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PHYSICS

Vol 1.

SZÉCHENYI 2020

Magyarország Kormánya

Európai Unió
Európai Szociális Alap

BEFEKTETÉS A JÖVŐBE

A tankönyv az EFOP-3.4.3-16-2016-00005 Korszerű egyetem a modern városban: Értékközpontúság, nyitottság és befogadó szemlélet egy 21. századi felsőoktatási modellben pályázat támogatásával készült.

Lektorálta: Simon Mihály
ISBN: 978-963-429-664-5
Kiadja: Pécsi Tudományegyetem Egészségtudományi Kar
Pécs, 2020.

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Introduction

Knowing the basic laws of nature has been, from our earliest history, the aspiration of mankind. It is known for the legacy of many prehistoric and ancient peoples, which have been proven to have extensive knowledge of the planets, stars, and the equinoxes of autumn and spring. The Arabs were able to predict the eclipse with great accuracy, the Greeks knew electricity and magnetism, possessed hydrostatic, static knowledge, the Romans built aqueducts, and all of them had amazing engineering and architectural knowledge. In the Middle Ages, the development of commerce brought with it an ever more accurate knowledge of the motion of celestial bodies, and it was at this age that the foundations of chemistry were also discovered. After the Enlightenment, the advancement of science was accelerated, and nowadays it has reached an unprecedented scale.

Physics is a science that speaks the language of nature. Physics seeks the fundamental laws that have shaped and moved the world around us since the beginning of the Universe 13.8 billion years ago.

Physics is an experiential science because it formulates its laws in an experiential way. We observe phenomena and try to reproduce them. In experimental circumstances, we always formulate laws based on the same phenomena and observations. We consider the law to be true until another phenomenon shows that it needs clarification, more general wording, or may need to be completely revised. This is called an inductive approach and is considered the most important tool in experimental physics.

Another way of knowing is the deductive method, the field of theoretical physics. Many discoveries have been made (mainly in the 19th and 20th centuries), in which, without experimental experience, laws have been derived theoretically from a basic assumption or truth. An excellent example of this is the existence of the electromagnetic waves predicted by James Clerk MAXWELL, or the theory of relativity described by Albert EINSTEIN, which was proved only after the theories appeared.

We use three different methods to formulate laws and regulations:

- The laws of nature are those we discover and apply regardless of man. The motion and functioning of any body or machine cannot contradict them.
- Axioms or polar truths (basic truths, basic facts) are basic principles that are taken for granted within the given framework and are not considered to require further proof. The exact definition of the axioms of science also means the birth of that science.

- The postulate is a requirement, a basic requirement, a basic assumption. It is similar to the axiom, but while the truth of the axioms is beyond doubt, it cannot be said with certainty about postulates.

By now Physics has learned these basic laws of nature in great detail. In observing the forces that drive nature, we come to know four kinds of interactions:

1. Strong interaction is the strongest one, for example it is responsible for attracting nuclei to protons and neutrons and has a very short range.
2. Electromagnetic interaction is an interaction of 10^{-2} times weaker than strong interaction between electrically charged particles with an infinite range.
3. Weak interaction is responsible for some phenomena on the atomic scale, such as radioactive β-decay. Its strength is less than that of electromagnetic interactions, only about 10^{-13} times the strength of the strong interaction, and its range is even smaller than that of the strong interaction.
4. The gravitational interaction between all particles and bodies. It is the weakest of the four basic interactions, about 10^{-38} times less potent than the strong interaction, but has an infinite range.

We distinguish between the disciplines of Physics, which include different phenomena with the same property, according to the subject matter of experimentation and study. These disciplines are:

Mechanics deals with the resting state of the body and its movements. The simplest approximation is to imagine bodies as mass points, and then construct points systems and expansive bodies to form their laws, taking into account that bodies are capable of deformation. Isaac NEWTON was the first to consolidate his fundamental laws in his book Philosophiae Naturalis Principia Mathematica (1687). Mechanics does not take into account the fact that the temperature of the bodies can vary and is not able to follow the motion of every element of a system with a very large number of mass points.

Describing processes where the temperature and, as a result, a characteristic of the body changes, is the task of *Thermodinamics*. This field of science examines what happens to different states of bodies when their thermal state changes, how their size, elasticity, or state changes. Also described in the topic of Thermodynamics is the description of the method of describing the statistical behavior of a system consisting of a large number of elements. We know the basic laws Thermodynamics as the Principle laws.

Electromagnetism deals with electrical and magnetic phenomena. It describes the interaction between electric charges and electric and magnetic fields. It's known only since the 19th century that electricity and magnetism are two manifestations of the same interaction and are most often present together. The unified system of the laws of electromagnetism is the four Maxwell equations. The first part of the Physics handbook covers these three disciplines.

The second part of the handbook is planned to deal with the following two topics.

The science of *Optics* explains the geometric propagation and wave nature of light. Light is considered a linear phenomenon when examined in a homogeneous and isotropic (non-directional) medium. If one wants to understand its nature, it can be explained by the propagation of electromagnetic vibrations in space and time, by electromagnetic waves. This provides an opportunity to discuss phenomena that can only be interpreted as waves.

Nuclear and Particle Physics discusses the interactions between the tiniest bodies. More than two millennia have passed since the ancient concept of the atom, due to the quest for the knowledge of the nature of matter. However, a real breakthrough was the explosive emergence of quantum mechanics in the early 20th century, a number of Nobel prize-winning ideas born. The equation describing the dynamics of the particle states is associated with the name of Erwin SCHRÖDINGER, which is considered the basic equation of quantum mechanics.

Astrophysics is the science of bodies of enormous size compared to us. It studies the formation, movement and changes of celestial bodies and deep space objects (nebulae, galaxies, etc.). For their description, Einstein's general theory of relativity proved to be most accurate. Because of the purpose of writing this book, this discipline is not included.

This book is for the Medical Diagnostic Analyst BSc program. It provides insights into the disciplines of Physics and the sub-areas that are required as basic knowledge in later professional subjects. In the course of writing the textbook, the physical quantity plays a central role, the survival of which is considered to be the basic law of our whole world: energy. We have endeavoured to avoid derivations that require a high level of mathematical knowledge or which are indispensable for the understanding and acquisition of the curriculum. We have tried to illustrate the theoretical considerations with examples.

In our nominations we used the following system:

- Physical quantities and units of measurement are written in italics, eg E (energy), 1 *m*.
- An arrow drawn above a physical quantity represents a vector quantity, eg: $\vec{r}$

- The mean value is given with the amount between triangle brackets, eg: $\langle P \rangle$

- A line drawn above the letters denoting the start and end points refer to the arc length or arc section, eg: $\overline{AB}$

- The multiplication operator (point) of the products in the context is written only in the case of the scalar product of the vectors, thus distinguishing it from the simple multiplication, eg: $\vec{F} \cdot \vec{r} = Fr\cos\alpha$

- For vectorial product of vector quantities, use the usual "cross" notation in mathematics, eg: $\vec{v} \times \vec{B}$ (read: vector v – cross –vector b).

Measurements are a basic criterion for determining physical quantities. A phenomenon can be described physically if the descriptive quantity can be measured. Measurement is a comparison with a predefined unit of measurement. The physical quantity is thus the product of the numerical value and the unit of measurement of the quantity.

The International System of Units, or SI (Système International d'Unités) for short, is a modern, internationally accepted system of units based on a few selected units of measure and their multiples or fractions expressed by different powers of 10. The SI system of measurement currently in use was adopted by the 11th General Conference on Weights and Measures in 1960. The basic quantities of SI and their standard units of measurement are as follows:

1. Length

The symbol for longitude is l, they are also used: s (path), r (radius), x, y, z (position coordinates), h (height), d (thickness or diameter), etc.

The base unit of length is the meter, which is equal to the path length that light travels in $\frac{1}{299\,792\,458}$ second in vacuum. Or in accordance with Zoltán BAY's 1983 recommendation: with 1 650 763.73 times the wavelength of radiation emitted by transitions between the $2p_{10}$ and $5d_5$ energy levels of a Krypton atom of mass number 86. Denoted by m.

2. Time

Time in physics is denoted by t (tempus), but τ (short duration) and T (period time) are also used.

The base unit of time is seconds, which is equal to the duration of the radiation corresponding to the transition between the two hyperfine energy levels of the 133 cesium atom. Denoted by *s*.

3. mass

Symbols of mass are *m* (massa) and *M* (great mass) and μ (small mass).

The unit of mass is the kilogram, as measured by the International Bureau of Weights and Measures. A roll of 90% platinum and 10% iridium in Sévres (near Paris) equals the mass of the international kilogram prototype. Denoted by *kg*.

4. Thermodynamic temperature

Thermodynamic or absolute temperature is denoted by *T* (temperature) and *t*, which is less frequently used (for other temperature scales).

The unit of measurement is kelvin, which is the $\frac{1}{273.16}$ of the thermodynamic temperature of the water triple point in honor of Sir William THOMSON, Lord of Kelvin. Denoted by *K*.

5. Electrical current

The electric current is denoted by *I*, but it can also be used with *i* (for alternating current).

The unit of measurement is the ampere. Current in two, straight, infinite length, and negligibly small circular cross-section conductor is 1 ampere, if a 1 *m* long part of them interacts with each other with 2 times 10^{-7} *N* in vacuum according to Ampere's law. Denoted by *A*.

6. Brightness

In physics, either the luminous intensity or the luminous flux is given the *I* signal (in fact, all flow rates are denoted as such).

The unit of measure is candela. 1 candela is the luminous intensity of $\frac{1}{6}10^{-5}m^2$ surface area of an absolute black body which radiates at the platinum's freezing temperature (2042 K) at 101 325 Pa pressure.

Such is the brightness of an ordinary wax candle, hence the name (candle). Denoted by *cd*.

7. Quantity of material

The quantity of material is denoted by n. It is measured in moles, which is the number of particles that is present in 0.012 kg of carbon (^{12}C). Denoted by *mol*.

Physics is the most exact science because it can use the language of mathematics to describe phenomena in the deepest way. Laws can be worded, but the mathematical context is the same for people of any nationality and mother tongue, without contradiction.

While reading the book, we wish our readers perseverance and success in learning the curriculum, and we wish him to love the science that is not created by, but wants to be known by man, and use it according to its purpose. Here's a final quote:

"Anyone who talks about the Planck constant and doesn't feel his voice trembling a bit did not understand anything!" - Edward Teller

Norbert Walter and Péter Kovács editors

1. Mechanincs

Mechanics is a discipline that studies the movement of bodies and changes in motion, their causes.

In the 20th century it was possible to conclude from the scientific results that our knowledge of natural laws and phenomena was not eternal. With the development of theories and the refinement of measurements, more general laws may replace or replace the old ones. Classic mechanics and relativistic mechanics are good examples. It can be shown that at speeds of magnitude lower than the velocity of light (such as the motion of macroscopic bodies), relativistic mechanics leads to the same results as classical (Newtonian) mechanics with great accuracy. Quantum mechanics also leads to the classical mechanical approximation of bodies of many orders of magnitude and mass greater than atomic size. In this section we only deal with classic mechanical problems, and within this we place more emphasis on point-like bodies with masses and material points.

1.1. Kinematics of the material point

1.1.1. Basic mechanical concepts, reference systems, location vector

To consider a body as a mass point (material point, point-like body) always means some approximation. The circumstances, the environment and the need for precision of the test will determine whether this approximation is permissible. Kinematics examines the characteristics of motion: where a mass point is at a given moment in a selected coordinate system, its speed and acceleration. We will express these in equations of motion. At the same time, we do not investigate the causes of motion, but do so later in dynamics (kinetics).

We always choose a coordinate system based on expediency, the one that best fits the given problem, where motion can best be described using mathematical tools. Imagine the coordinate system fixed to an object. What we attach to the coordinate system basically influences the nature of the motion in it, the equations that characterize it. For example, in a straight-line train with smooth movement, its wheels perform a smooth circular motion in the coordinate system fixed to the train. A point on the same wheel moves from a coordinate system fixed to Earth on a cyclic path. An object dropped by a person in the train moves in a straight-line, accelerating motion (freefall) in the coordinate system fixed to the train, while in the coordinate system fixed to the Earth it moves in a parabolic path (horizontal throw).

1.1.1.1. Cartesian Coordinate System

In describing the motions we will use the Cartesian Coordinate System (Fig. 1.1), whose axes are x, y, z, and the unit vectors are $\vec{i}$, $\vec{j}$, $\vec{k}$ from the origin (O). (the horizontal arrow above the letter indicates the vector quantity).

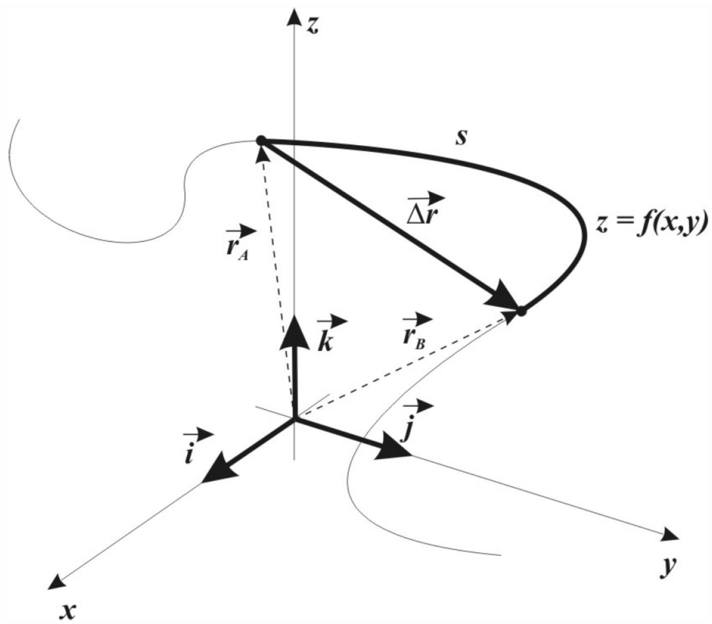

Figure 1.1

In this system, each of the three coordinate axes is perpendicular to the other two, each of the three coordinates represents a given distance from the origin, and *x*, *y*, and *z* follow each other as the thumb, index, and middle fingers of the right hand stretched out perpendicularly in pairs.

The location of the center of mass (*P*) is characterized by the location vector $\vec{r}$, whose endpoint pointing to *P* is generally progressing a function of a time curve, called a space curve. The location vector $\vec{r}$ can be written as the result of component vectors pointing to the axes:

$$\boxed{\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}}$$

where $x(t), y(t)$ and $z(t)$ are the time-dependent components of the absolute value of the location vector $\vec{r}(t)$, or so-called trajectories.

This means that the motion of the center of gravity of P – in this reference system – is the superposition of three perpendicular linear motions.

The function $\vec{r}(t)$ is mathematically a so-called one-parameter (the parameter it the *t* time) vector-scalar function to which mathematics can be applied.

The absolute value (length) of $\vec{r}(t)$, ie $|\vec{r}(t)|$, can be calculated from the Pythagorean theorem.

At a given moment *t*:

$$|\vec{r}(t)| = \sqrt{x^2(t) + y^2(t) + z^2(t)}$$

1.1.1.2. Polar coordinate systems

In addition to the Cartesian coordinate system, the plane and spherical polar coordinate system can be used to provide location and calculate physical quantities derived from location coordinates. Their use is justified in cases where the symmetry of the movement makes it easier to calculate with them.

The planar polar coordinate system can be used when the motion of a given point P is circularly symmetrical. Here, the position of the P point is characterized by the distance r from the origin and the angular rotation φ relative to a given reference direction. The value of the radius r can be between 0 and $+\infty$, and the polar angle φ can be any value between $+\infty$ and $-\infty$ (Figure 1.2).

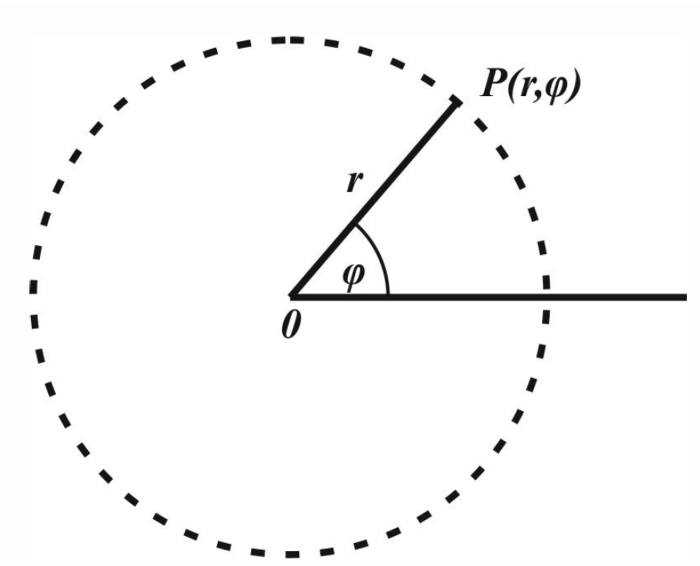

Figure 1.2

One of the most commonly used systems for spatial motion or for solving a problem with spherical symmetry is the spherical polar coordinate system. They are also used in the geographical coordinate system. In this case, the location of a given P is characterized by three coordinates: distance or radius r from the origin, horizontal angle φ or azimuth relative to the reference direction (this is the longitude on Earth) and vertical angle υ or polar angle relative to the reference direction (this is the latitude on Earth, Figure 1.3).

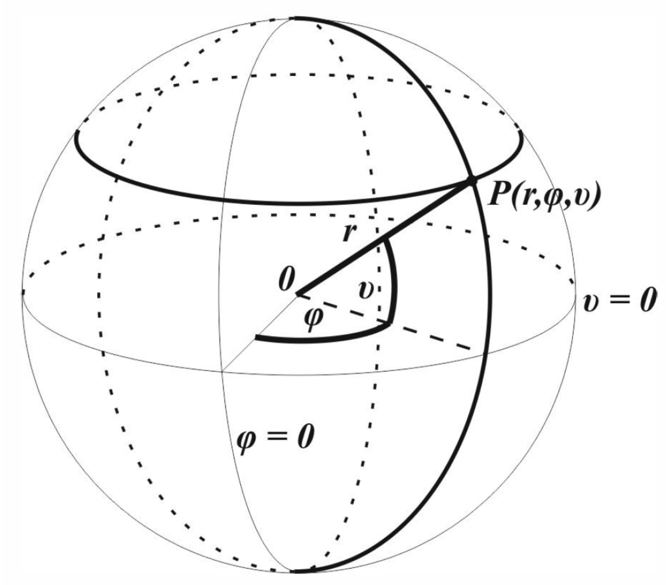

Figure 1.3

1.1.2. Velocity

Let at time t the point of mass to be at the point P_1 of the trajectory with the location vector $\vec{r}(t)$, after $\Delta t > 0$ at point P_2 with the location vector $\vec{r}(t+\Delta t)$ at time $t+\Delta t$. Then the displacement of the material point during Δt is:

$$\Delta\vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t)$$

During this time, as shown on Figure 1.4. the point runs the length $\overline{P_1P_2}$ of the whole or part of the trajectory, called the $s \equiv \overline{P_1P_2} \geq |\Delta\vec{r}|$ path. Thus, the path s is equal to the absolute value of the displacement in the case of a straight line or less than that of a curved line.

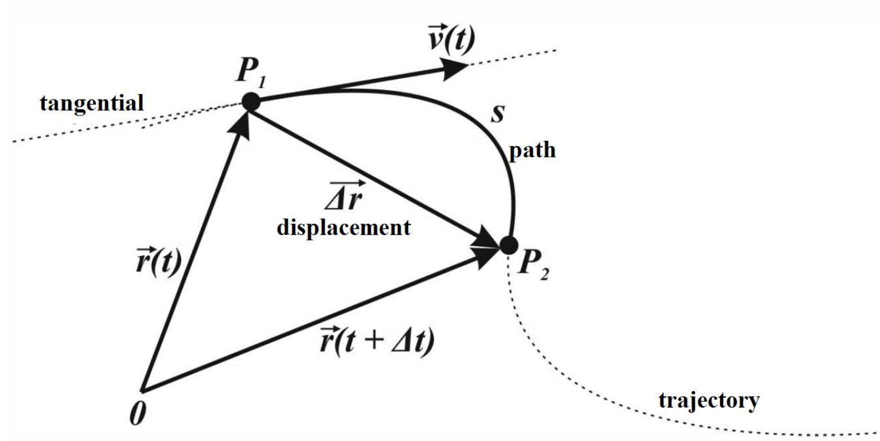

Figure 1.4

The quotient $\frac{\Delta\vec{r}}{\Delta t}$ is called the average velocity (pointing towards $\Delta\vec{r}$), by decreasing Δt closing to the instantaneous velocity at point P_1, whose exact value is

$$\boxed{\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}(t)}{\Delta t} = \frac{d\vec{r}(t)}{dt} = \dot{\vec{r}}(t)}$$

and its numeric value shows how much constant velocity the body would go if the velocity change would not occur at that moment.

Comment: The "point" above the quantity, (read $\Delta\dot{\vec{r}}(t)$ as r-point) is to distinguish time derivation, and we will continue to follow this notation system.

Thus, the instantaneous velocity vector is the first time derivative of the location vector $\vec{r}(t)$. The $|\dot{\vec{r}}(t)|$ absolute value of this is the magnitude of the speed:

$$v = |\dot{\vec{r}}(t)| = |\vec{v}(t)| = \sqrt{v_x^2 + v_y^2 + v_z^2}\,.$$

By its speed definition, its SI unit is the meter per second, signified by: $\frac{m}{s}$. A frequently used non-SI unit is $\frac{km}{h}$, the conversion is: $1\frac{m}{s} = 3.6\frac{km}{h}$.

1.1.3. Acceleration

The acceleration vector (short for acceleration) is the rate of change of the velocity vector, and its numerical value shows the change in velocity per unit time. By definition

$$\boxed{\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}(t)}{\Delta t} = \frac{d\vec{v}(t)}{dt} = \dot{\vec{v}}(t) = \ddot{\vec{r}}(t) = \ddot{x}(t)\vec{i} + \ddot{y}(t)\vec{j} + \ddot{z}(t)\vec{k}}$$

where $a_x = \ddot{x}(t)$, $a_y = \ddot{y}(t)$ and $a_z = \ddot{z}(t)$ are the scalar components of the acceleration vector.

Thus, the acceleration vector is the first time derivative of the velocity vector and the second time derivative of the location vector. Its direction is the same as the direction of the $\Delta\vec{v}(t)$ velocity change and its magnitude is:

$$a = |\vec{a}(t)| = \sqrt{a_x^2 + a_y^2 + a_z^2}\,.$$

By definition, its SI unit is meters per square second, ie $\frac{m}{s^2}$

If the mass point is moving in a straight line, the direction of its $\vec{v}(t)$ velocity vector and its $\vec{a}(t)$ acceleration vector will be in the same straight line. In this case, it is advisable to record the coordinate system so that the movement is on one of the axes. This simplifies the task of calculating scalar quantities.

In the case of non-linear motion, the acceleration vector does not lie in a straight line with the velocity vector, but at a certain angle (Figure 1.5).

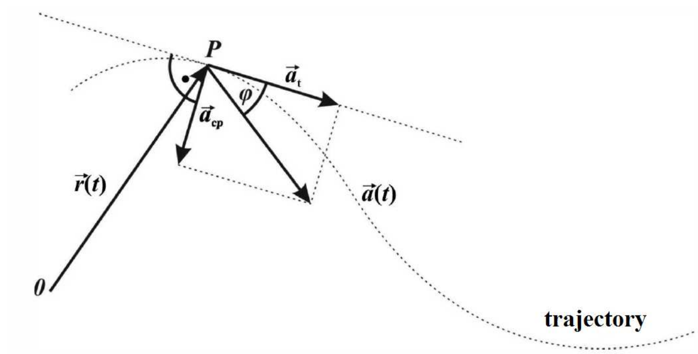

Figure 1.5

The plane stretched by the $\vec{v}(t)$ velocity vector and the $\vec{a}(t)$ acceleration vector is called the fitting plane of the *P* point of the trajectory. The $\vec{a}(t)$ acceleration vector can then be resolved into a $\vec{a}_t(t)$ tangent (tangential or so-called trajectorial component) and a perpendicular to it and to the velocity vector, or so-called normal direction $\vec{a}_n(t)$ component:

$$\boxed{\vec{a}(t)=\vec{a}_t(t)+\vec{a}_n(t)}.$$

1.1.4. Free fall, gravitational acceleration

Experience has shown that a body that is freely released close to the Earth's surface, regardless of its mass, moves at constant acceleration to the Earth (as far as drag in the air is waived), also known as free fall. The acceleration is called gravitational acceleration, which is distinguished by *g*. The gravitational acceleration is a vector quantity, its direction is toward the center of gravity of the Earth (vertically down). It is easy to see that the value of *g* depends on the height from the earth's surface (in fact, the distance from the earth's center of gravity), but it is not so obvious that it is also on the latitude. On the geographical latitude of Hungary *g* is around 9.81 $\frac{m}{s^2}$.

1.1.5. Horizontal throw

The motion of the body thrown horizontally at an initial velocity v_0 is examined in the horizontal x direction and in the vertical z direction. Because its motion varies in the z direction (free fall) and not in the x direction, its composite motion is determined by the trajectories of the $x(t)$ and $z(t)$ components in the two directions (Figure 1.6).

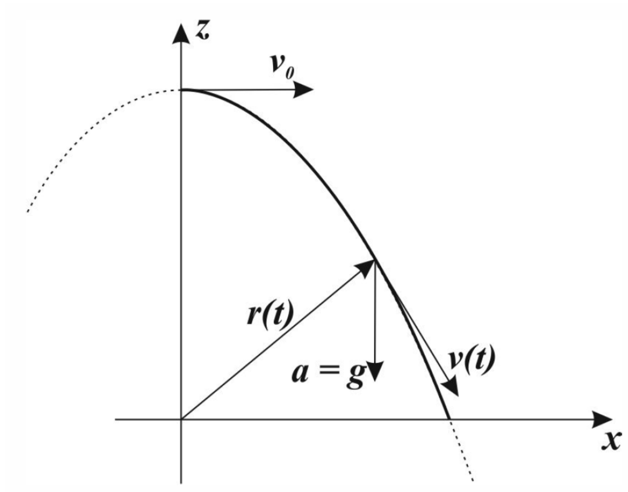

Figure 1.6

1.1.6. Smooth circular motion

Smooth circular motion is defined as the trajectory of the body being circular and running in equal intervals of arc by equal time intervals. The relationships of circular motion can be easily understood by drawing an analogy between linear smooth motion and smooth circular motion.

When talking about s path at linear motion, it is the angular rotation φ from a given starting position at circular motion. It has a unit SI in radians, however, it is designated by 1.

The circular motion is characterized by the T circulating time or period time, which is the time required to complete a full circle.

The reciprocal of the period time, ie the number of laps per unit time, is n number of rotations. From the definition it is clear that $n = \frac{1}{T}$, its SI unit is $\frac{1}{s}$ (actually $\frac{rotations}{s}$).

For linear motion, the distance travelled per unit time is numerically equal to the velocity v, which for circular motion is equivalent to the angular rotation per unit time ie. to the ω angular velocity. Its SI unit is $\frac{rad}{s} = \frac{1}{s}$. The angular velocity can be calculated as the quotient of the angular rotation and the associated duration. For a complete lap, that is 2π radians, and it takes T time, thus:

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi n}.$$

Comment: In fact, angular velocity is also a vector quantity whose direction is upright perpendicular to the plane of circular motion when it is counterclockwise, downward when it is clockwise.

For straight line motion, the path is calculated $s = vt$, and for circular motion, the angular rotation is $\varphi = \omega t$.

The acceleration of smooth linear motion is zero, since the speed does not change. In this, however, the circular motion differs from it, because in this case the direction of the velocity vector changes. Acceleration is present all the time when any characteristic (either magnitude or direction) of the velocity vector changes. While the magnitude of the velocity vector is constant, its direction varies at any given moment. So smooth circular motion has a normal acceleration called centripetal acceleration:

$$\boxed{a_{cp} = r\omega^2}$$

whose direction points to the center of the circle at all moments.

Comment: The quantity analogous to the acceleration defined in the case of uniformly changing linear motion is the so-called angular acceleration in the case of uniformly changing circular motion: β. Its SI unit is $\frac{1}{s^2}$, and it is true that:

$$\beta = \frac{\Delta\omega}{\Delta t}.$$

The product of the r radius of the orbit and the angular speed ω is the circumferential velocity:

$$v_k = r\omega.$$

Note that for circular motion, we used the planar polar coordinate system to describe the motion because of the circular symmetry, instead of the Cartesian coordinate system. The radius r of a circle is constant, the angle φ is a polar angle whose magnitude depends on time. Of course, circular motion can also be described by Cartesian coordinates, but in this case the description would be much more complicated, since the temporal changes of both coordinates can be specified by sine and cosine functions:

$$x(t) = r\cos(\omega t)$$

and

$$y(t) = r\sin(\omega t)$$

while in polar coordinates the equations of motion simplify to:

$$r = constant$$

and

$$\varphi(t) = \omega t\ .$$

If the circular motion does not start from the horizontal but from the initial angular rotation of φ_0, then the functions in Cartesian coordinates would be:

$$x(t) = r\cos(\omega t + \varphi_0) \text{ and } y(t) = r\sin(\omega t + \varphi_0).$$

The above is explained is illustrated in Figure 1.7.

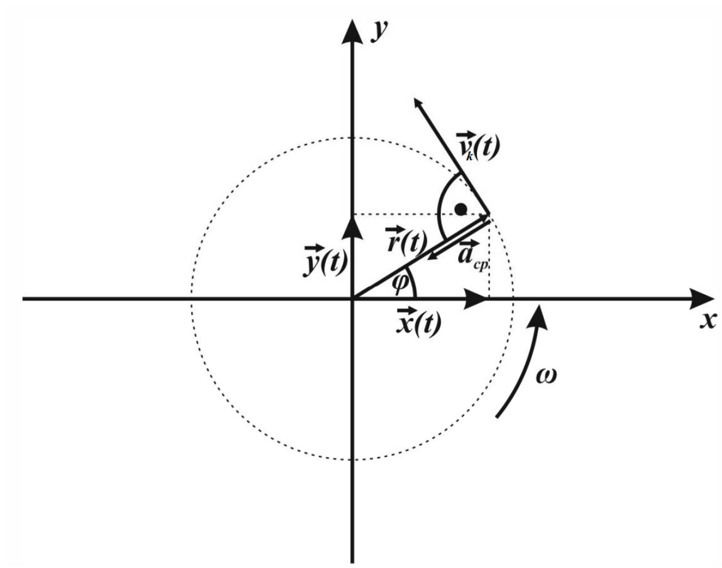

Figure 1.7

1.1.7. Harmonic vibration

Mechanical vibrations are periodic changes in the state of a material point or of a material system of mass points.

It’s called harmonic vibration when the location-time function of motion is a sine or cosine function. The following shows that the smooth circular motion projected onto a wall perpendicular to the plane of the circle produces a harmonic vibration. Ideally, a spring-mounted body will perform the same movement (see below).

1.1.7.1. displacement-time function

Let us examine Figure 1.8, in which a point-like body P performs a circular motion of angular velocity ω (or period T and n number of rotations) on a path of radius r.

This motion is projected horizontally on a wall perpendicular to the circular plane, where P' is the shadow of the body. Find the current displacement of P' as a function of time.

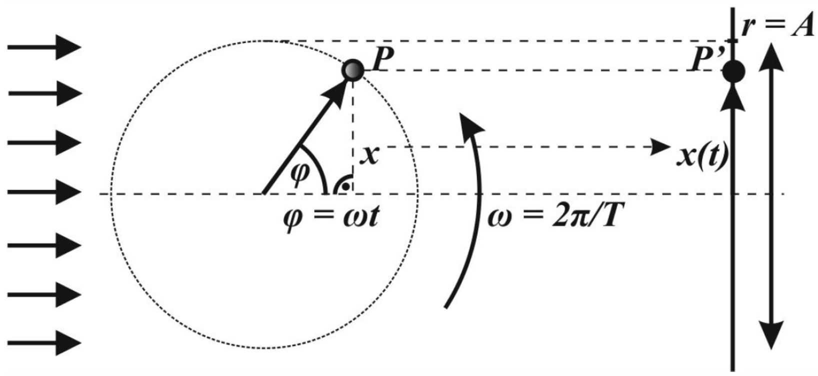

Figure 1.8

To determine the location, you need a starting point (origin), which in this case is the position of the horizontal position of the circular motion, the so-called equilibrium position. The equilibrium situation is presented in Figure 1.8, it is a horizontal dashed line. In this case, the instantaneous position of P' can be characterized by the distance from the equilibrium position, which is called the x displacement. The displacement changes from moment to moment, but its values are periodically repeated, its maximum being the amplitude A, the maximum distance from the equilibrium position. It can be seen that the amplitude is equal to the radius of the circle, ie: $A = r$.

The value of the displacement is the same as the horizontal segment x drawn at the P point and the equilibrium position, which is drawn in the circular motion. The right triangle shows that

$$x = r\sin\varphi$$

Because the circular motion is smooth, so

$$x(t) = r\sin(\omega t)$$

Considering the above, the displacement-time function of the vibration is replaced by $A = r$:

$$\boxed{x(t) = A\sin(\omega t)}$$

Comments:

1. If the circular motion does not start from the horizontal but from an initial angle of φ_0, then the function is $x(t) = A\sin(\omega t + \varphi_0)$

2. The vertical projection would be $y(t) = A\cos(\omega t + \varphi_0)$*, so the circular motion can be split into two, as a result of harmonic vibrations perpendicular to each other.*

1.1.7.2. phase angle

For circular motion, φ and ω are visible physical quantities: angular rotation and angular velocity. But what is their meaning in harmonic vibration, which is a linear motion, so talking about this angle and angular velocity is not fortunate, but they are still included in the expression. To do this, we illustrate the displacement-time function (Figure 1.9):

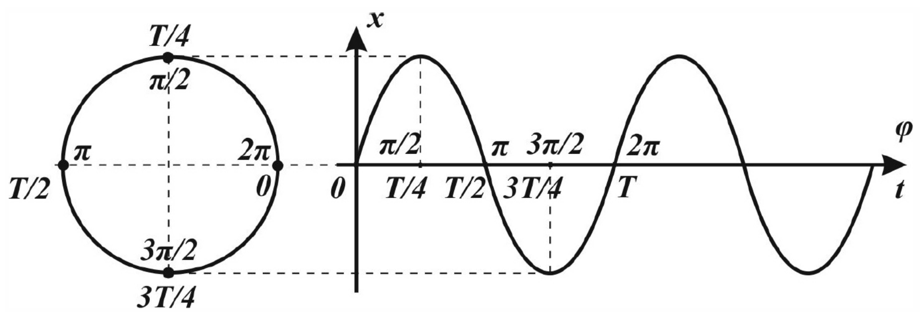

Figure 1.9

Time is plotted on the horizontal axis. In this case, a complete vibration takes place in the time it takes for the circular motion to complete a full circle (T). However, it is also possible to plot the displacement as a function of φ instead of time, since φ and t are proportional to each other ($\varphi = \omega t$). In this case, any momentary state of vibration corresponds to an amount of angular dimension whose numerical value is equal to the instantaneous angular rotation of the circular motion, but does not mean "angle" but the vibration state or phase. According to this, phase 0 corresponds to equilibrium position, 2π corresponds to maximum displacement. When the vibration first returns to equilibrium, its phase is π, but this vibration state is not the same as phase 0 because its velocity is in the opposite direction from the initial position (in the case of circular motion, P is on the opposite side). The vibration state will again be the same as the initial state when the circular motion point has completed a complete circle, i.e. its angular rotation is 2π, while at $\frac{3\pi}{2}$ P' again has maximum deviation in the opposite direction.

Based on the above, we define two concepts:

1. Two vibrations or states of one vibration are called the same when phase difference is:

$$\Delta\varphi = (2n-2)\pi$$

where $n = 1,2,\ldots$ Thus, in such cases the phase difference may be 0, 2π, 4π, etc.

2. Two vibrations or states of one vibration are called opposed when phase difference is:

$$\Delta\varphi = (2n-1)\pi$$

where $n = 1,2,\ldots$ Thus, in such cases the phase difference may be π, 3π, 5π, etc.

ω angular velocity for circular motion numerically equals to the rate of change in angular rotation. In vibration, the same numerical value represents the rate of change of the vibration phase, called circular frequency. Calculated as:

$$\omega = \frac{2\pi}{T},$$

its SI unit is $\frac{1}{s}$ ($\frac{rad}{s}$ in fact).

1.1.7.3. Velocity-time function

There are two ways to derive the velocity-time function of harmonic vibration: in analogy to the displacement geometrically, or by differentiation.

1.1.7.3.1. geometrically

Geometric derivation has three things to consider:

1. The instantaneous velocity vector $\vec{v}(t)$ of the vibration equals to the vertical component of the circumferential velocity vector of the circular motion $\vec{v}_k$ (Fig. 1.10).
2. Circumferential velocity and angular velocity are related to each other: $v_k = r\omega$
3. The angle of angular rotation and the enclosed angle of the circumferential velocity vector to vertical direction are angles of perpendicular stem, ie they are equal.

Based on these, it is easy to see that the instantaneous velocity of the vibration is:

$$\boxed{v(t) = v_k \cos\varphi = v_k \cos\omega t = r\omega\cos\omega t = A\omega\cos\omega t}$$

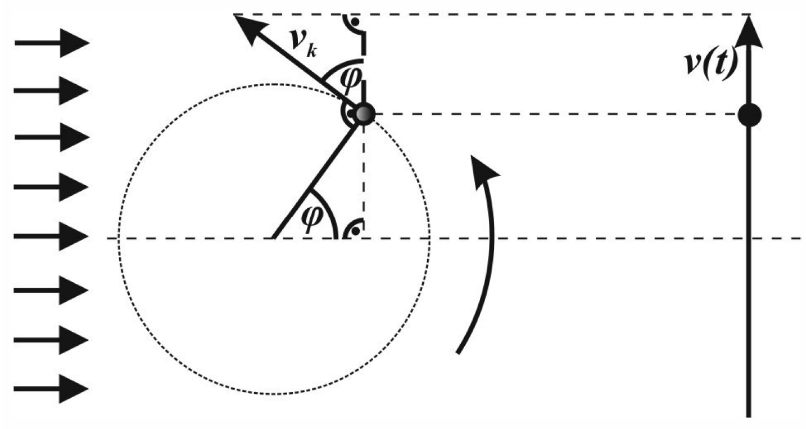

Figure 1.10

1.1.7.3.2. differentiation

The velocity-time function is given by the time-derivative of the displacement-time function, so:

$$v(t) = \dot{x}(t) = A\omega \cos \omega t$$

Comment:

When the initial angle is φ_0, then the function is $v(t) = A\omega \cos(\omega t + \varphi_0)$

In the moments when the oscillatory moving body moves through the equilibrium position ($x = 0$), its velocity is maximal at:

$$v_{\max} = A\omega,$$

and when its displacement is maximum, ($x_{\max} = A$), its velocity is zero.

1.1.7.4. Acceleration-time function

The acceleration-time function can also be derived geometrically and by differentiation.

1.1.7.4.1. geometrically

The instantaneous acceleration of the harmonic vibration is the vertical component of the centripetal acceleration of the circular motion (Figure 1.11).

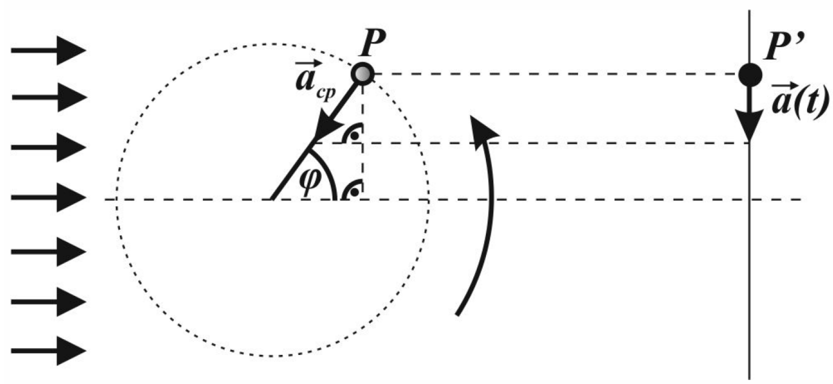

Figure 1.11

Since circular motion has been discussed in a polar coordinate system, and we know that the position of the *P* point can be given by a location vector, it is important to note that the centripetal acceleration vector is always opposite to the location vector *r*:

$$\vec{a}_{cp} = -\vec{r}\omega^2 .$$

The enclosed angle of the centripetal acceleration vector with the horizontal direction equals the instantaneous angular rotation (φ), therefore:

$$\boxed{a(t) = a_{cp}\sin\varphi = a_{cp}\sin\omega t = -r\omega^2\sin\omega t = -A\omega^2\sin\omega t = -\omega^2 x(t)}.$$

1.1.7.4.2. differentiation

The acceleration-time function is obtained by deriving the velocity-time function:

$$a(t) = \dot{v}(t) = -A\omega^2\sin\omega t = -\omega^2 x(t) .$$

Comments:

1. The negative sign in the acceleration-time function indicates that the instantaneous acceleration vector is always in the opposite direction to the instantaneous displacement vector.

2. When the initial angle is φ_0, then the function is $a(t) = -A\omega^2\sin(\omega t + \varphi_0) = -\omega^2 x(t)$

By plotting the acceleration-time function (Fig. 1.12) and comparing it with the displacement and velocity graphs, it can be seen that acceleration is maximal (value: $A\omega^2$) when both the displacement is maximum and the instantaneous velocity is zero. It's zero in moments when the displacement is zero and the instantaneous velocity is maximum.

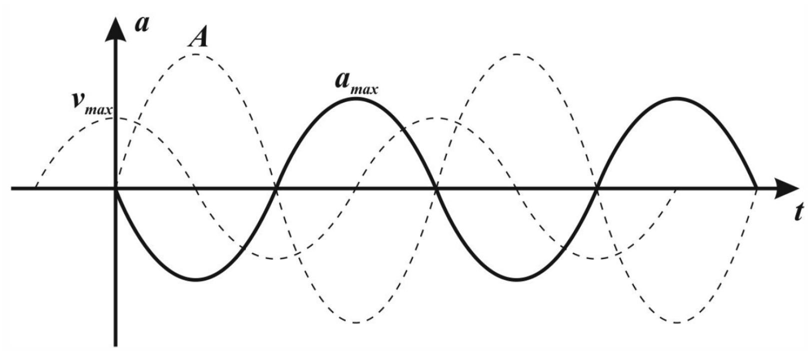

Figure 1.12

1.1.8. Composition of harmonic vibrations

As a result of two or more harmonic oscillatory movements, harmonic and non-harmonic (anharmonic) vibrations and even non-periodic movements can occur. In special cases, these movements are well illustrated and can be handled by elementary mathematical tools. We only give examples of special cases.

1.1.8.1. Harmonic vibrations of the same frequency in a straight line composition

Let two equations of motion describe the harmonic vibration of a line:

$$x_1(t) = A_1 \sin(\omega t)$$

and

$$x_2(t) = A_2 \sin(\omega t + \varphi_0)$$

Then the resulting displacement is given by the sum of x_1 and x_2:

$$x(t) = x_1(t) + x_2(t) = A_1 \sin(\omega t) + A_2 \sin(\omega t + \varphi_0).$$

By mathematical transformations its:

$$x(t) = A \sin(\omega t + \varphi)$$

where A and φ can be calculated from the given data. We can state that the resultant motion is also a harmonic vibration. Special cases of this are the following.

1.1.8.1.1. Amplification

If $\varphi_0 = 0$, the resulting displacement is:

$$x(t) = (A_1 + A_2)\sin(\omega t)$$

that is, the equation of motion of a harmonic vibration whose amplitude is the sum of the amplitudes of the component vibrations, which are mutually amplify.

1.1.8.1.2. Attenuation

If $\varphi_0 = \pi$, the vibrations are always in the opposite phase and therefore:

$$x(t) = (A_1 - A_2)\sin(\omega t)$$

that is, the amplitude of the resulting harmonic vibration is the difference between the components, the vibrations are attenuating each other.

1.1.8.1.3. Extinction

If $A_1 = A_2$, then $x = 0$, the vibrations extinct each other.

1.1.8.1.4. Floatation

If the vibrations are not of equal frequency, there is a great variety of resulting movements. We consider the case when $A_1 = A_2$ and $\varphi_0 = 0$. In this case

$$x(t) = A(\sin\omega_1 t + \sin\omega_2 t)$$

which can by shaped by trigonometric transforms as

$$x(t) = 2A\sin\left(\frac{\omega_1 + \omega_2}{2}t\right)\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)$$

This is a periodic movement, but not harmonious. It is a special case when $\omega_1 \approx \omega_2$. Then, with the introduction of $\omega = \frac{\omega_1 + \omega_2}{2}$, the resulting evasion:

$$x(t) = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)\sin\omega t$$

which represents a sinusoidal vibration of ω with a time-dependent amplitude $2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)$. However, this amplitude changes very slowly over time relative to the amplitude of sinusoidal vibration $\frac{\omega_1 - \omega_2}{2} << \omega$. This phenomenon is called floatation (Figure 1.13). For example, if two forks of nearly the same frequency are played at the same time, this amplification and fading of the pulsating sound can be well observed by the human ear.

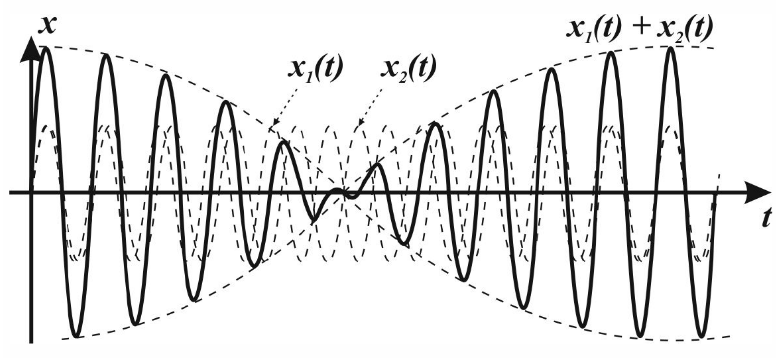

Figure 1.13

1.1.9. Anharmonic vibrations

Vibrations which cannot be described purely by a single sine or cosine function are called anharmonic vibrations.

1.1.9.1. Damping vibrations

It has been mentioned that, ideally, a spring-hooked body also performs a harmonic oscillatory movement. However, the ideal case does not exist because certain influencing circumstances cannot be ruled out.

These can be:

- medium resistance, friction
- the spring mass is not zero
- the spring develops heat due to deformation.

Because of these influencing conditions, the vibrating body loses energy during movement, which means that energy is dissipated. The damping vibration is described by the

$$\boxed{x(t) = A_0 e^{-\beta t} \sin \omega t}$$

motion equation. This equation differs only in the $e^{-\beta t}$ factor from the harmonic vibration displacement-time function, which causes the initial A_0 amplitude to decrease exponentially. It can be seen from the graph (Figure 1.14) that the function $e^{-\beta t}$ is the envelope of the damped vibration graph.

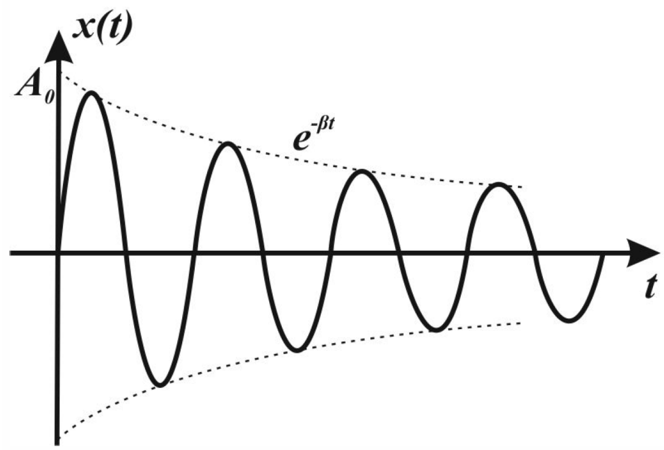

Figure 1.14

The degree of damping is determined by the damping factor β in the exponential exponent. On the basis of β we can talk about weakly attenuated, strongly attenuated or over-damped vibrations. The frequency, which is determined by the physical properties of the system (the weight of the vibrating body and, for example, the coefficient of elasticity of the spring), is the so-called self-frequency, usually denoted by f_0.

1.1.9.2. The Fourier theorem

Joseph FOURIER described how a complex signal can be described as an infinite sum of simple sine vibrations. Fourier used this principle to describe heat dispersion by solving a series of differential equations. According to the Fourier theorem of mathematics, any periodic but non-harmonic (anharmonic) vibration can be described as an infinite series of harmonic (sine or cosine) vibrations:

$$f(x) = \frac{c_0}{2} + a_1 \cos \omega x + a_2 \cos 2\omega x + \ldots + a_n \cos n\omega x + b_1 \sin \omega x + b_2 \sin 2\omega x + \ldots + b_n \sin n\omega x$$

where a_i and b_i show a decreasing tendency. The name of ω is the fundamental frequency (or basic harmonic), whose integer multiples are the harmonics.

Figure 1.15 illustrates with the first two terms of the Fourier series how to approximate a y-axis symmetric rectangular vibrations by cosine functions. As more members are added to the queue, the result will give a rectangular vibration with less and less error.

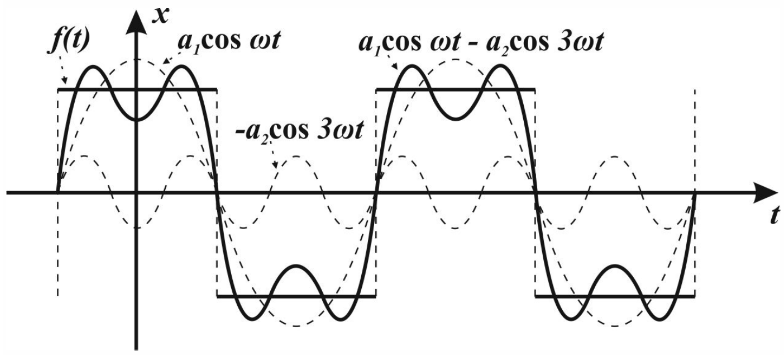

Figure 1.15

The frequency distribution of a compound vibration is called a spectrum. The spectrum can be continuous if the frequencies follow each other continuously without a jump, and linear (discrete) if only certain frequencies occur in it and they do not follow each other continuously. For example, the spectrum of the rectangular vibration shown in Figure 1.16 contains frequencies with decreasing amplitude.

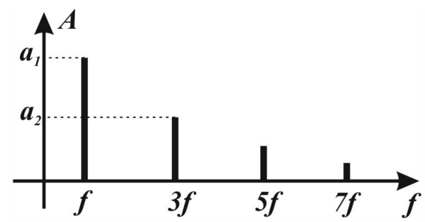

Figure 1.16

1.1.10. Mechanical waves

When the vibration center (the excitation vibration or wave source) that produces the mechanical vibrationg is surrounded by a flexible medium, the vibration energy can propagate from point to point. The propagation of vibration from mass point to mass point in space and time is called a mechanical wave. Wave propagation is

- spatial, because the wave goes from point A to point B,
- temporal because it takes some time for the wave to travel this distance.

1.1.10.1. Physical characteristics of mechanical waves

Time period:

The time distance between two closest points of the same vibration phase, or in other words the shortest time it takes for a given point to enter the same vibration state, is the T period of the wave. This necessarily corresponds to the period of the wave source. The SI unit is the second, denoted by s.

Frequency:

The number of waves generated per unit time is the frequency f of the wave, which is numerically equal to the frequency of the wave excitation oscillator. Its SI unit is $\frac{1}{s} = Hz$ (hertz).

Amplitude:

The maximum displacement of the vibrating points is called the amplitude A. The units of measurement in SI are meters: m.

Wavelength:

The spatial distance of two closest points of the same vibration phase, equal to the path traveled by the wavefront during T, is the wavelength λ of the wave. Its SI unit is the meter, sign: m.

Propagation velocity:

Since the waves move at a constant velocity in a given homogeneous medium, the propagation velocity of the waves in the waves is:

$$\boxed{c = \frac{s}{t} = \frac{\lambda}{T} = \lambda f}$$

From the relation it can be seen that the frequency and the wavelength of a given velocity are inversely proportional to each other. For the same propagation velocity, a high frequency has a small wavelength and a low frequency has a large wavelength. Unit of propagation velocity in SI: $\frac{m}{s}$

1.1.10.2. Waveform comparison of media

If the propagation velocity of the two media is the same, the two media are said to be waveform identical, otherwise they are different waveforms. For example, if the propagation velocity in Medium 1 is greater than that in Medium 2 ($c_1 > c_2$), Medium 1 is rarer and Medium 2 is denser. There may be two different media that are wave-identical. For example, if the propagation velocity of a mechanical wave is the same for two different materials, they are wave-identical. However, there may be different media made of two identical materials, if they have different physical characteristics. For example, the speed of sound wave propagation in the air is influenced by the air temperature. In this case, two layers of air at different temperatures are waveform different. The same phenomenon is observed in the case of waves spreading over the water surface. The velocity of the water wave propagation depends on the depth of the water. In such cases, the deeper water is waveform different from the shallower one.

1.1.10.3. Classification of mechanical waves

1.1.10.3.1. In the direction of the displacement

- A wave is called transversal if the displacement is perpendicular to the direction of propagation.
- A longitudinal wave is considered when the displacement is parallel to the direction of propagation.

1.1.10.3.2. By the dimension of the medium

- One-dimensional (1D) waves that form along a line. With good approximation, these are waves created on a stretched rubber rope.
- Two-dimensional (2D) waves that form on a surface. For example, waves on the water surface.
- Three-dimensional (3D) waves that are formed in space. Such is the sound wave or earthquake shock waves.

1.1.10.4. Wave equation and wave function

Due to its mathematical complexity, the equation describing the formation of mechanical waves in a flexible medium is not derived. The wave equation describing the movement of an x directional disturbance in the medium in the perpendicular to the y propagation direction (transversely) is a partial differential equation, which is:

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}}$$

where c is the propagation velocity of the wave. The solution to this equation is any function that satisfies the condition that the original disturbance arrives at the x-coordinate location after $\frac{x}{c}$ time. Mathematically:

$$y(x,t) = y\left(t - \frac{x}{c}\right)$$

This general mathematical solution offers ample opportunity to physically solve the wave equation. The number of solutions in which waves initiated by any shape, any excitation vibration or pulse, and traveling at a velocity c along the x-axis is practically infinite. Of the many solutions, however, the important for our discussion are the

$$y(x,t) = A\sin\omega\left(t - \frac{x}{c}\right)$$

shaped harmonic wave functions. The waveform along the line is depicted at time t after departure from the origin in Figure 1.17:

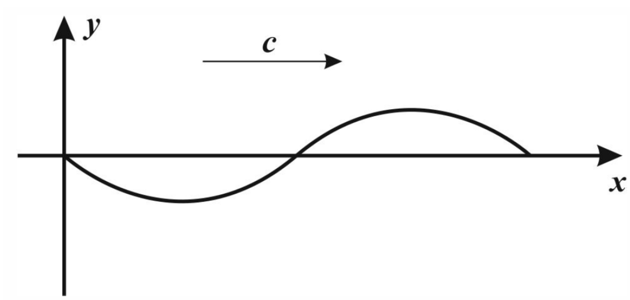

Figure 1.17

A point x away from the origin produces a harmonic vibration whose phase is $\frac{x}{c}$ delayed over time relative to the excitation vibration phase of the source at the origin.

Comments:

1. The symbol ∂ is used to denote a derivative of a single variable in a quantity dependent on several variables, the so-called partial derivative.

2. In case of spatially propagating waves the wave equation from the $\psi(\vec{r},t) = \psi(x,y,z,t)$ wave function is

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$$

The wave function resulting from the solution of a homogeneous isotropic sphere wave:

$$\psi(\vec{r},t)=\frac{A_0}{r}\sin\omega\left(t-\frac{x}{c}\right),$$

where A_0 is the surface amplitude of the sphere wave in the square length dimension, which is numerically equal to the amplitude of the wave at a unit distance from the source. Thus, the surface amplitude decreases inversely with increasing distance. This wave function describes a wavelet phenomenon with a good approximation at much greater distances from the source compared to the size of the source.

1.1.10.5. Properties of waves

Waves have 5 fundamental properties: reflection, refraction, deflection, interference and polarization. The first two properties, reflection and refraction, occur when the wave reaches the boundary of two media that are different waveform, $c_1 \neq c_2$. These two properties can be represented geometrically by considering only the property of the wave that it propagates in a homogeneous medium. In this case, the waves are represented by a straight line along their propagation direction.

1.1.10.5.1. Reflection

The wave arriving at the boundary is reflected from the boundary. The perpendicular to the fluid boundary is the so-called incident perpendicular (Figure 1.18). Experience shows that the incident angle α enclosed by the incident wave and the incident perpendicular and the reflection angle α' enclosed by the reflected wave and the incident perpendicular are the same:

$$\boxed{\alpha=\alpha'},$$

and the incident wave, reflected wave, and incident perpendicular are in the same plane.

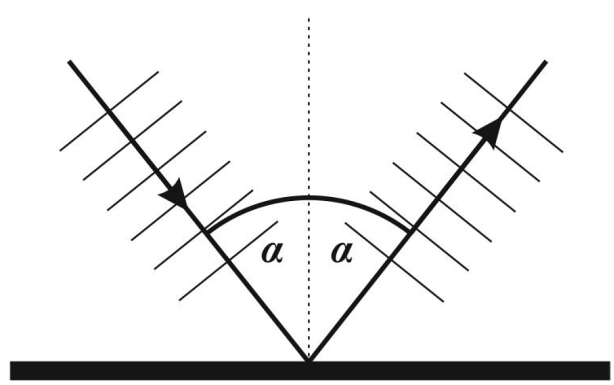

Figure 1.18

1.1.10.5.2. Refraction

When the wave crosses a media boundary and the two media are differently waveformed ($c_1 \neq c_2$), the travel direction of the wave is changed (Figure 1.19). The law of experiential refraction of waves is the Snellius-Descartes law, which states:

"The sine of the incident angle α enclosed by the incident wave and the incident perpendicular is proportional to the sine of the refraction angle β enclosed by the forward wave (refracted) and the incident perpendicular, as wave velocities measured in the two media."

That is, mathematically:

$$\boxed{\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2}}$$

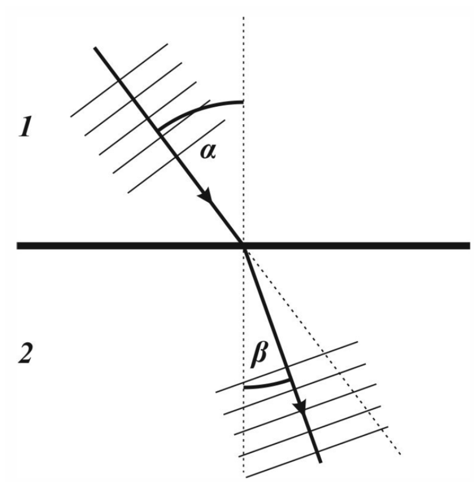

Figure 1.19

Since c_1 and c_2 are constant in a homogeneous medium, their ratio is also constant. The name of this constant is the refractive index of Medium 2 relative to Medium 1, in short (n_{21}), so

$$n_{21} = \frac{c_1}{c_2} = \frac{1}{n_{12}}$$

Cases of wave refraction include:

1. From a rarer medium to a denser medium

If the wave passes from the rarer medium 1 to the denser medium 2, i.e. $c_1 > c_2$, the refractive index n_{21} will be greater than 1, so the refractive angle will be smaller than the incident angle. For this we say that the wave refracts *towards* the perpendicular incident (Figure 1.19).

2. From a denser medium to a rarer medium

If the wave passes from the denser medium 1 to the rarer medium 2, i.e. $c_1 < c_2$, the refractive index n_{21} will be less than 1, so the refractive angle will be greater than the incident angle. For this we say that the wave refracts *from* the perpendicular incident (Figure 1.20).

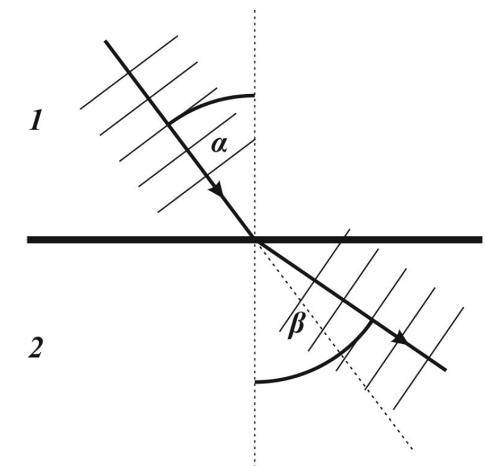

Figure 1.20

3. The angle of total reflection

In the former case, there is a certain incident angle at which the refractive angle is exactly 90º. This angle is called the α_0 limit angle of the total reflection. At this angle, there is still a refraction, but if the wave is with any bit larger incident angle than the limit angle (the dashed line in Figure 1.21), then the law of reflection applies.

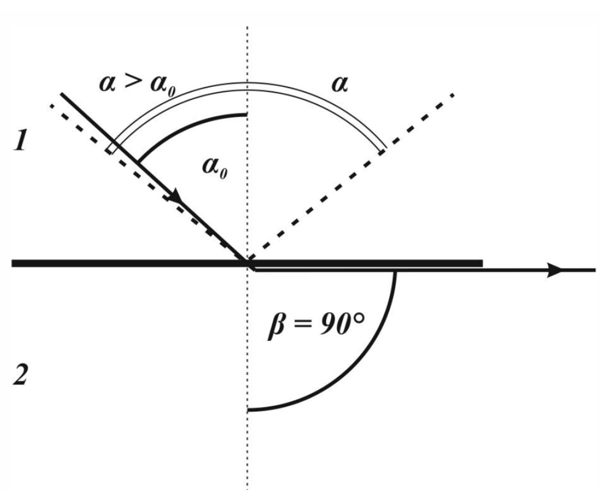

Figure 1.21

Then the following is true:

$$\frac{\sin\alpha_0}{\sin\beta} = \frac{\sin\alpha_0}{\underset{(=1)}{\sin 90^\circ}} = \sin\alpha_0 = n$$

4. Waveform identical media

If $c_1 = c_2$ (the two media are waveform identical) then the incident angle and the refraction angle are the same, ie no refraction occurs.

The other three characteristics of waves, deflection, interference and polarization, are no longer geometrically explained, and must be described in terms of the characteristics of the waves.

1.1.10.5.3. Diffraction

Diffraction occurs when a gap of width d is placed in the path of the wave. A typical wave diffraction pattern is illustrated in Figure 1.22. Then we would expect the wave source and the gap (or obstacle) to geometrically define a so-called umbra where the wavefronts would not appear. However, experience shows that they also appear here, ie they lean towards the umbra, and the greater the degree of inclination, the smaller the gap (or barrier) size (Figure 1.22). Finally, if the diameter d of the gap (or obstacle) is commensurate with the wavelength λ, then the diffraction extends over the entire umbra (Figure 1.22), so that the gap and the obstacle behave as an elementary wave source.

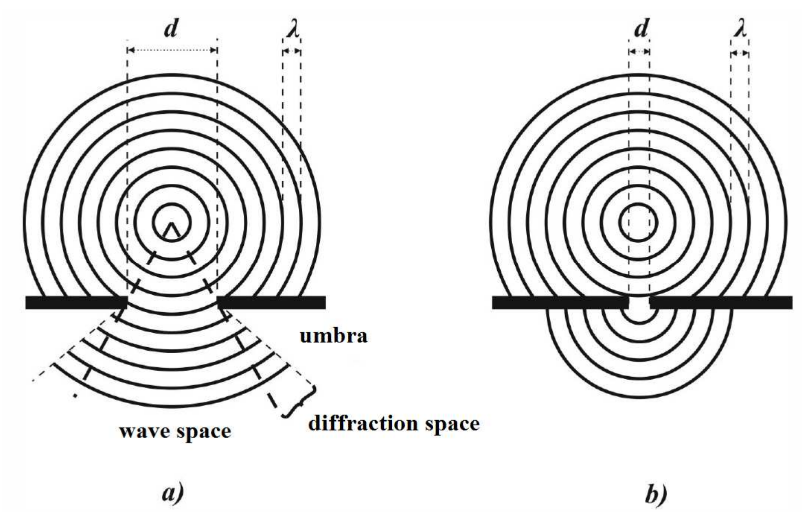

Figure 1.22

The experiential law of diffraction can be interpreted on the basis of the Huygens-Fresnel principle:

"Each point of the wave space is an elementary wave source. The waveform at time dt is given by the interference of the elementary waves starting at moment t."

1.1.10.5.4. Interference

When two or more waves meet, the waves add up, which is called interference, and the result is a new waveform. The displacement of the resulting wave at the point of intersection of the resulting interference wave is obtained by summing the instantaneous displacement of the two interfering waves at a given point vectorially. The specific cases of interference thus generated, according to the results obtained in the composition of unidirectional vibrations, are as follows

1. Amplification

Amplification occurs when the two meeting waves are of the same frequency and phase:

$$f_1 = f_2$$

$$\varphi_1 - \varphi_2 = (2n - 2)\pi$$

where $n = 1, 2, \ldots$ in this case the phase difference is 0, 2π, 4π, etc. (Figure 1.23)

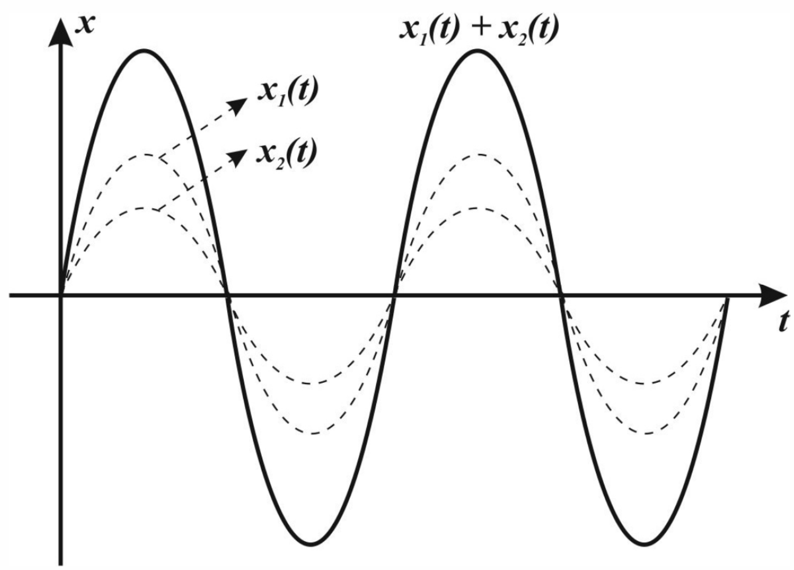

Figure 1.23

2. Attenuation

Attenuation occurs when the waves have the same frequency but encounter opposite phase and have different amplitudes:

$$f_1 = f_2$$

$$A_1 \neq A_2$$

$$\varphi_1 - \varphi_2 = (2n-1)\pi$$

where $n = 1, 2, \ldots$ in this case the phase difference is π, 3π, 5π, etc. (Figure 1.24)

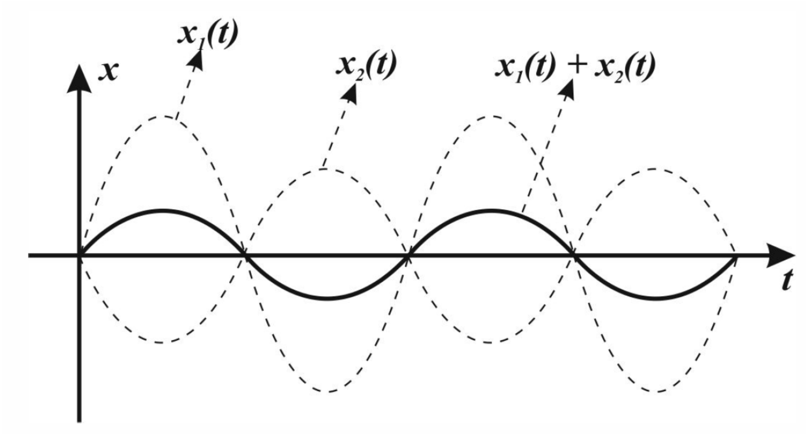

Figure 1.24

3. Extinction

An extinction occurs when two waves of the same frequency, opposite phase but of the same amplitude are interfering:

$$f_1 = f_2$$

$$A_1 = A_2$$

$$\varphi_1 - \varphi_2 = (2n-1)\pi$$

where $n = 1, 2, \ldots$ in this case the phase difference is π, 3π, 5π, etc. (Figure 1.25)

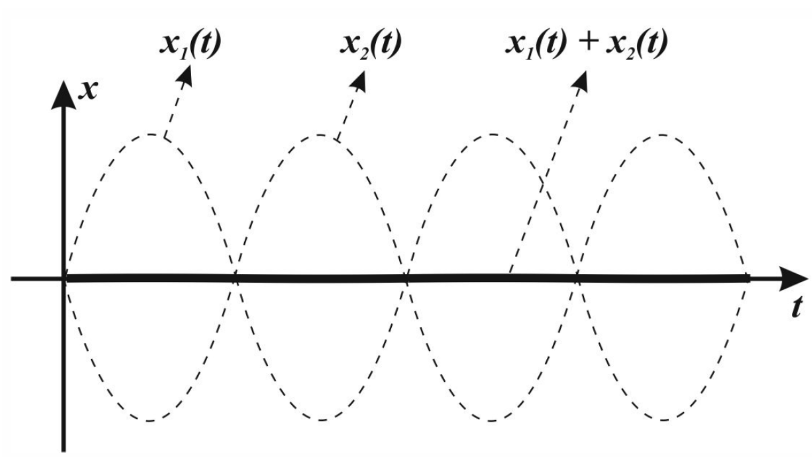

Figure 1.25

4. Floating

Floating is a special case of interference when two waves of nearly the same frequency ($f_1 \approx f_2$ or $\omega_1 \approx \omega_2$) meet. Then an $\frac{\omega_1 + \omega_2}{2}$ circular frequency is generated, sometimes with a higher amplitude, that is, with maximum amplification, and sometimes with a smaller amplitude, i.e., a maximum attenuated wave, in which the rhythm of amplification and attenuation is $\frac{\omega_1 - \omega_2}{2}$ (Fig. 1.13).

1.1.10.5.5. Polarization

There are waves that do not have a particular direction of vibration, these are the unpolarized waves. Waves that vibrate in a particular direction of vibration (e.g., vertical) are called polarized waves.

To produce a polarized wave, for example, place a so-called diaphragm, e.g. a motion limiter parallel gap, in the path of the unpolarized wave. (Fig. 1.26) In this case, the diaphragm passes through only vibrations in the direction parallel to it, it filters out all other vibrations. The result will be a polarized wave whose polarization plane is parallel to the diaphragm. Adding another diaphragm perpendicular to the previous one (i.e. the polarization plane) in the path of the polarized wave then no longer allows the vibrations which are perpendicular to it to pass, so that the wave is extinguished.

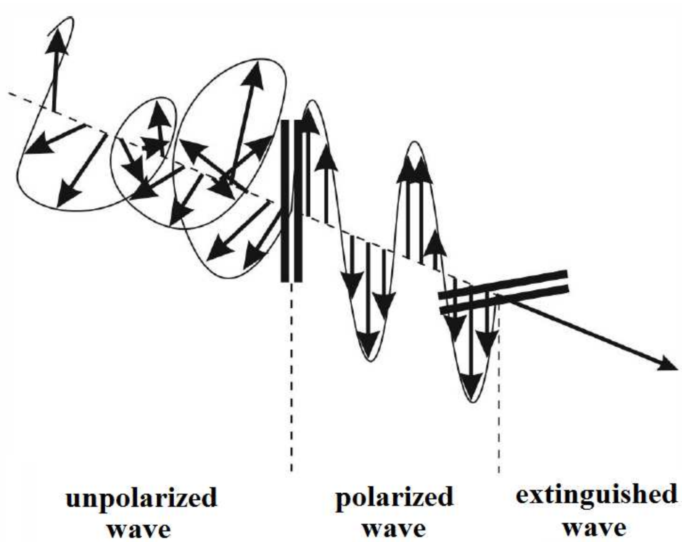

Figure 1.26

The mode of polarization discussed above is plane polarization. However, polarization is defined as any case in which the vibrations of the wave occur according to a particular symmetry. This is the case, for example, with a circularly or elliptically polarized wave, in which the vertices of the deflection vector of consecutive points are projected on a circle or ellipse projected perpendicular to the direction of travel.
According to the property of polarization, only transverse waves can be polarized.

If a phenomenon has the properties of waves, then we can think of it as a wave phenomenon. The same applies to mechanical waves, but as we will see, it needs to be overridden in the case of electromagnetic waves.

1.1.10.6. The Fermat principle

The laws of reflection and refraction theoretically can also be deduced from the Fermat principle which states that:
"The waves travel between two points on the path that requires the smallest amount of time."

This is the principle of minimum time. From the corresponding equation $t'(x) = 0$, the laws of reflection and refraction can be derived theoretically.

1.2. Dynamics of the material point

Dynamics is also called as science of force, because its basic concept is force as the cause of movements. Thus, in dynamics, we study the cause motions, and in a special case the conditions of calm. That's how we talk about kinetics and Statics. Statics examines the physiological conditions for lasting calm (balance). We will see that we will rely on the concepts we are familiar with in kinematics and that we will also need to introduce new concepts and quantities in kinetics.

1.2.1. The Laws of Kinetics of the Material Point

1.2.1.1. Newton's axioms

1.2.1.1.1. Newton's 1^{st} axiom (under the laws of Galileo GALILEI and Johannes KEPLER)

The main task of Kinetics is to investigate the causes of motion and to determine $\vec{r}(t)$, $\vec{v}(t)$ and $\vec{a}(t)$ motion equations. Experience has shown that the velocity of an abandoned body is constant. If the body's velocity changes, there is always a body (or bodies) in its environment that cause the velocity to change. Related to this is Newton's first axiom, the law of inertia:

“Each body maintains its rest or straight line motion until another body or field forces it to change”

The law states that it is a general observation that the state of motion of the bodies can be altered only by an external effect, which in itself are incapable of doing so, that is to say they are powerless. The physical quantity introduced for the degree of inertia is the mass (more precisely, the inert mass), denoted by *m* and its unit in SI is *kg*. Newton's 1^{st} axiom is a universal law.

The question is in which reference system this axiom applies, since speed depends on the choice of the reference system (coordinate system). Experience has shown that the axiom is true with high accuracy in the coordinate system fixed to the standing stars. Reference systems in which the law of inertia applies are called inertial systems (inertia systems). A coordinate system that performs linear motion with respect to the inertial system is also an inertial system.

Earth with radius $R = 6.37 \cdot 10^6 m$ and rotation period $T = 8.64 \cdot 10^4 s$ is considered to be an inertial system because its centripetal acceleration resulting from its rotation, for example. on the equator is only:

$$a_{cp} = R\omega^2 = R\frac{4\pi^2}{T^2} = 0.034\frac{m}{s^2}$$

which is negligibly small.

Newton's assumption is that the motions are always related to a uniform, motionless and body-independent, so-called "Absolute space", it proved to be incorrect.

1.2.1.1.2. Newton's 2^{nd} axiom

The physical effect that changes the movement state of the interacting body is called force. Force is the vector quantity representing the magnitude of the force effect, denoted by F.

Comment:

In the case of a non-point body, the interaction may also take the form of deformation, and the extent of the force effect may be defined on this basis.

The change of motion state, i.e. the change of velocity $\vec{v}(t)$, is characterized by the acceleration $\vec{a}(t)$.

Based on this, Newton's 2^{nd} axiom states that:

"The force acting on the body and the acceleration of the body are linearly proportional and unidirectional."

That is, mathematically:

$$\vec{F} \sim \vec{a}$$

From the linear proportionality that force is constant multiplication of acceleration, that is

$$\boxed{\vec{F} = m\vec{a} = m\ddot{\vec{r}}}$$

where m is a constant characteristic of the body, the inert mass of the body.

The term "inert mass" refers to the degree of "resistance" the body exerts on the force that changes its state of motion. In other words, a body that is less accelerated by the same force is more inert.

Comment:

In the case of homogeneous bodies, the mass is directly proportional to the volume, ie $\frac{m}{V}$ is *constant. This constant is the density of the body, denoted by ρ, thus:* $\rho = \frac{m}{V}$, *SI unit:* $\frac{kg}{m^3}$.

According to the defined units of acceleration and mass, the unit of force in SI can be derived: $k\frac{m}{s^2} = N$ (newton), that is, 1 N is the force that moves a 1 kg body at 2 $\frac{m}{s^2}$ acceleration.

Comment:

Earth moves its bodies at acceleration 9.81 $\frac{m}{s^2}$*, so that it exerts a 9.81 N force on a 1 kg body.*

In equation $\vec{F} = m\vec{a} = m\ddot{\vec{r}}$, $\vec{F}$ is generally the relationship between the point of mass, and some obvious function of the common position, velocity, etc. of the interacting body, so-called Force Function. The calculation of the equations of motion is possible by solving the differential equation in the knowledge of the force function acting on the case.

1.2.1.1.3. Newton's 3rd axiom

Newton's 3rd axiom states that:

“The interacting bodies (A and B) exert equal and opposite forces on each other.”

So mathematically:

$$\boxed{\vec{F}_{A,B} = -\vec{F}_{B,A}}$$

This axiom is also called the law of action-reaction. We use the concepts of *force* to describe one act, and *counter-force* to describe the counteract. It is important to note that one force acts on one body while the other force acts on the other body.

Any force that results from the interaction of two bodies in an inertial system has a counter-force. However, there is no counter-force for forces which, due to the change of motion state of a non-inertial system, are only detectable in the given reference system. Such is the centrifugal force acting on the observer in the rotating reference system.

1.2.1.1.4. Newton's 4th axiom

The body may be subject to multiple forces at the same time.

“The forces can be vectorialy summed and their combined effect can be replaced by their equivalent result vector, that is, the forces acting on the body can be replaced by a single resultant force.”

The magnitude of the resulting force $\sum \vec{F}$ is:

$$\boxed{\sum \vec{F} = \sum_{i=1}^{n} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n}$$

This is what usually called Newton's 4th axiom or independent superposition of forces.

1.2.1.2. The basic equation of dynamics

Combining Newton's 2nd and 4th axioms is the basic equation of dynamics. It calculates the equations of motion of any body by describing the acceleration-time function $\vec{a}(t)$. This describes the change in the state of the motion function by knowing the magnitude and force functions of the forces acting on the body. By integrating this, given its initial conditions, we obtain the $\vec{v}(t)$ velocity-time function. Another time integration results in the location-time function $\vec{r}(t)$ of the body. The basic equation for dynamics is:

$$\boxed{\sum \vec{F} = m\vec{a} = m\ddot{\vec{r}}}$$

1.2.1.3. Forces and force laws

The relationships that give strength according to direction and magnitude as a function of the environment and the characteristics of the body are called force laws. Knowing the force laws and Newton's axioms, the motion of bodies can be described.

1.2.1.3.1. Gravitational force, gravity and weight

1. A gravitational force F_g is exerted by the Earth with mass M on a gravitational object of mass m, located at a distance r from the Earth's center of gravity, and is:

$$\boxed{F_g = G\frac{Mm}{r^2}},$$

where $G = 6.67428 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$ the universal gravitational constant and the direction of the force pointing to the center of gravity of the Earth.

2. The gravitational force acting on and near the Earth's surface is called gravity, and here its value is due to its relative constancy of $g = G\frac{M}{r^2}$:

$\vec{G}_g = m\vec{g}$

3. Horizontally held or hung bodies on Earth, at rest relative to it, exert a F_{press} compressive or F_{pull} tensile force on the holder or rope. According to Newton's 3rd axiom, holder or rope exerts a force at equal amount, but in the opposite direction of F_{hold} holding or F_{r} rope force on the body. The dynamic condition of rest is that the forces acting on the body balance each other, so that the holding or rope force exerted by the holder or suspension is equal to, but opposite to, the gravitational force acting on the body (Fig. 1.27).

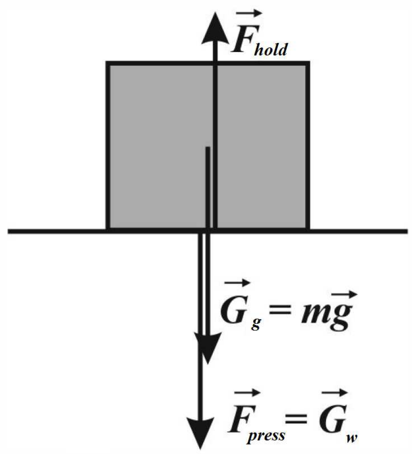

Figure 1.27

Therefore, starting from the example of a held body:
From the condition of rest:

$$\sum \vec{F} = 0$$

that is

$$\vec{G}_g = -\vec{F}_{hold} \,,$$

and their magnitude:

$$\left|\vec{G}_g\right| = \left|\vec{F}_{hold}\right| = mg \,.$$

According to Newton's 3rd axiom:

$$\vec{F}_{press} = -\vec{F}_{hold}$$

that is

$$\left|\vec{F}_{press}\right| = \left|\vec{F}_{hold}\right| .$$

It follows from the comparison of the two equations that the held or hung bodies exert a force of magnitude

$$\boxed{\left|\vec{F}_{press}\right| = \left|\vec{G}_g\right| = mg}$$

on the holder or rope.

The compressive or tensile force F_{press} and F_{pull}, expressed in mg by the body of mass m for horizontal holder or rope, is called the weight force G_{w} (in short: weight) with its point of contact at the geometric center of the contacting surfaces, pointing towards the Earth's center of gravity.

Free falling bodies are affected only by gravity. In the absence of a holder, the body does not exert any compression, ie its weight is zero (weightless).

1.2.1.3.2. Slip friction force

Experience has shown that there is a slip frictional force F_{s} between contacting displaceable bodies, independent of the relative velocity of the bodies up to a few $\frac{m}{s}$, and in good proportion to the compression force F_{press} perpendicular to the contacting surfaces:

$$\boxed{F_s = \mu F_{press}},$$

where μ depends on the material quality of the contacting bodies. Name: slip coefficient of friction, which is a ratio without a unit. The direction of the F_{s} slip frictional force acting on the body is opposite to the direction of motion of the sliding surface.

Comment:

The counter-force of slip friction force is the opposite effect of the body on the ground.

1.2.1.3.3. Adhesion friction force

The effect of interfering surfaces preventing the movement of bodies relative to one another is adhesion friction, which is a measure of the adhesion friction force. Maximum value:

$$\boxed{F_{a,\max} = \mu_0 F_{press}},$$

where μ_0 is the coefficient of adhesion friction, the ratio without a unit.

When the magnitude of the pulling force acting on the body is greater than the maximum adhesion frictional force, the body moves. μ_0 is greater than μ, e.g. $\mu \approx 0.8$. and $\mu_0 \approx 0.9$ for tires on asphalt. Due to the adhesion frictional force, we can walk and vehicles can move on Earth.

1.2.1.3.4. Spring force

Ideally, the spring, stretched or compressed on length x, exerts a tensile or compressive force F_{sp}, proportional to the elongation or compression of x, and always in the opposite direction. This is Hooke's Law, which applies to the limit of flexibility:

$$\boxed{\vec{F}_{sp} = -D\vec{x}},$$

where D is the so-called. spring constant, which shows how much force the spring will have per unit of elongation, measured in units of: $\frac{N}{m}$.

1.2.1.3.5. Medium resistance force

Our environment (eg air, water) exerts a force of medium resistance on bodies moving relative to it, which impedes the movement of the body. Experience has shown that the magnitude of the medium resistance force F_m depends on the density of the medium, the shape and the surface of the body, and at low velocity the relative velocity $\vec{v} = \dot{\vec{x}}$ of the medium and the body:

$$\boxed{\vec{F}_m = -k\vec{v} = -k\dot{\vec{x}}}$$

where k is the so-called medium resistance coefficient, which is constant depending on the density of the medium, the shape and the surface of the body.

1.2.1.3.6. Compressive force and pressure

The bodies exert a compressive force on the holding surface. By definition, the quotient of the compression force F_{press} (or component of the compression force perpendicular to the surface) and the surface A is referred to as pressure, denoted by: p. calculation:

$$\boxed{p = \frac{F_{press}}{A}}.$$

The SI unit of pressure is $\frac{N}{m^2} = Pa$ (pascal), denoted as Pa. Normal atmospheric pressure is 101 325 $Pa \approx 10^5$ Pa.

1.2.1.3.7. Hydrostatic compressive force and pressure

Let a liquid of height h with a density ρ_l in a vessel with bottom surface A. The liquid exerts a compressive force on the bottom of the vessel equal to the weight of the liquid (Figure 1.28).

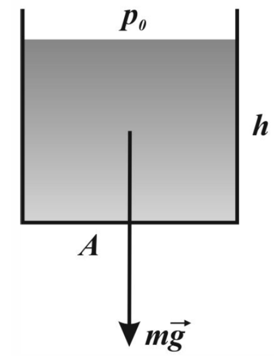

Figure 1.28

The pressure exerted by the liquid on the bottom of the vessel due to the compressive force is the so-called. hydrostatic pressure. According to the definition of pressure:

$$\boxed{p = \frac{F_{press}}{A} = \frac{mg}{A} = \frac{\rho_l V g}{A} = \frac{\rho_l A h g}{A} = \rho_l h g}.$$

Under the Pascal Law:

"External pressure exerted on liquids, due to its incompressibility, continues to expand unattenuatedly to all directions in the fluid."

If the surface of the liquid is subjected to an external pressure p_0 (eg air pressure), the pressure in all directions at depth h from the surface is:

$$p = p_0 + \rho_l h g .$$

1.2.1.3.8. Buoyancy force

Archimedes recognized that:

"Bodies immersed in fluids are affected by a buoyancy force equal to the weight of the fluid displaced by the body"

The Archimedes' Law is one of the oldest laws in science.

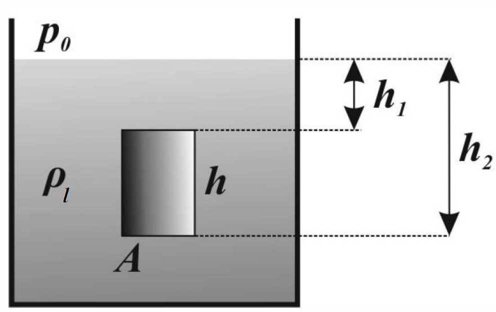

Figure 1.29

As shown in Figure 1.29, the forces exerted on the sidewalls of a submerged column body are zero, and the forces exerted on the lower and upper faces are:

$$F_b = F_2 - F_1 = p_2 A - p_1 A .$$

To replace the expression above for hydrostatic pressure:

$$F_b = (p_0 + \rho_l h_2 g)A - (p_0 + \rho_l h_1 g)A = \rho_l g (h_2 - h_1) A$$

Since $(h_2 - h_1)A = V_b$ is the volume of the body equal to the volume of fluid displaced by the body, it can be seen that the F_b buoyancy force exerted upward by the hydrostatic pressure on the body is equal to the weight of the liquid displaced by the body. Its magnitude is:

$$\boxed{F_b = \rho_l V_b g}$$

and indeed equal to the $\rho_l V_b g = m_l g$ weight of the displaced liquid.

1.2.1.5. Impulse (momentum, amount of movement)

The product of the mass *m* and the velocity *v* of a body is called impulse (momentum or amount of motion) and is denoted by *I*:

$$\boxed{\vec{I} = m\vec{v}}.$$

The impulse is vector quantity, its direction is the same as the velocity direction. SI unit: $kg\frac{m}{s}$.

Newton formulated his 2nd axiom using this quantity, that

$$\vec{F} = \frac{d\vec{I}}{dt} = \dot{\vec{I}}$$

or:

$$\vec{F} = \frac{d(m\vec{v})}{dt},$$

which is called impulse theorem. If *m* is constant (according to classical mechanics) then:

$$\frac{d\vec{I}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F} .$$

It can be seen that the impulse change is:

$$d\vec{I} = \vec{F}dt ,$$

also called "push". If $F = 0$, then $dI = 0$, that is, if there is no force on the body (or if the sum of forces is zero), the impulse of the body does not change, but it is constant. This is called the law of impulse retention:

“In the absence of external forces, the momentum of the closed system is constant.”

1.2.1.6. The torque

Bodies fixed to the axis can rotate under the influence of force if the line of action of the force does not coincide with the axis of rotation. It matters how far the force line is from the axis of rotation and also has a different rotational effect if its direction is different. To describe the rotational effect of the force, we introduce the concept of torque. The torque of force $\vec{F}$ on the axis of rotation at point O is given by the

$$\boxed{\vec{M} = \vec{r} \times \vec{F}}$$

vector, where $\vec{r}$ is a vector pointing from point O to the point of attack of force $\vec{F}$. Torque value is:

$$M = rF \sin\alpha ,$$

where $r \sin\alpha$ is the distance of the line of action of the force from point O, in short: k lever. The torque is thus:

$$M = kF$$

the product of force and leverage, the unit in SI being the Newton meter, denoted by Nm.

1.2.1.7. The angular momentum

The vectorial product of the instantaneous impulse vector $\vec{I} = m\vec{v}$ with the $\vec{r}$ instantaneous radius of the trajectory of a non-linearly moving body of mass m is the angular momentum of the body, denoted by $\vec{L}$, calculated as:

$$\boxed{\vec{L} = \vec{r} \times \vec{I}},$$

its direction is specified by the right hand rule. The magnitude is

$$L = rI \sin\alpha = rmv \sin\alpha ,$$

where α is the angle enclosed by velocity $\vec{v}$ and radius $\vec{r}$ (Figure 1.30):

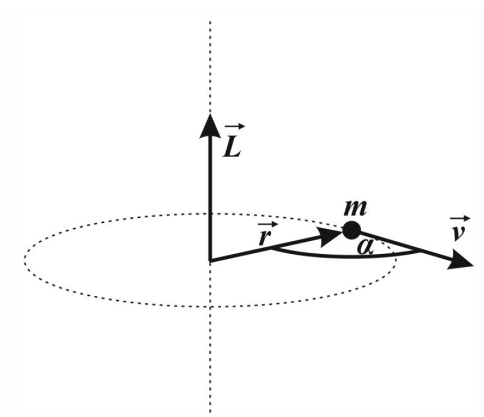

Figure 1.30

It can be shown that there is a similar relationship between the angular momentum and the torque acting on the body as between the force and the impulse:

$$\vec{M} = \frac{d\vec{L}}{dt} = \dot{\vec{L}}\ .$$

“If the sum of torques of the forces acting on the body is zero, then the change in the angular momentum of the body is also zero, that is, the angular momentum is constant, remaining quantity. ”

This is called the law of angular momentum retention.

1.3. Work and energy

1.3.1. The energy

Interacting bodies are capable of modify one of the physical characteristics of the other. This property is called changeability. The physical quantity introduced to characterize changeability is energy. Energy is a scalar quantity that characterizes material systems.

"In a closed system isolated from its surroundings, the value of energy remains constant over time."

This is the law of energy conservation. The law was formulated by many scholars for special cases, and the general modern acceptance of the principle stems from the publication of the German physicist Hermann von HELMHOLTZ in 1847.

1.3.2. The work

When the energy of the body changes by organized motion, impulse (receiving and transferring), it is called working, and the energy transmitted by working is called work, sign: W.

Based on these the theorem of mechanical work is:

$$\boxed{W = \Delta E},$$

i.e.

“A work on a body thermally insulated from its surroundings results in the energy change of the body.”

Working in present in Physics if the acting force (or the resultant of the forces) has a perpendicular component to the displacement of the body (Figure 1.31).

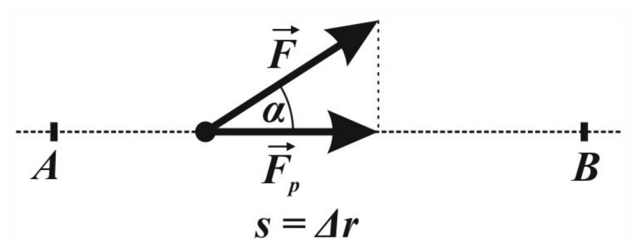

Figure 1.31

For constant magnitude and direction force and displacement on a straight line, work is the product of the force component $\vec{F}$ in the direction of displacement $\Delta\vec{r}$ and the displacement, which is mathematically a scalar product of the force $\vec{F}$ and the displacement vector $\Delta\vec{r}$.

$$W = \vec{F} \cdot \Delta\vec{r}$$

The SI unit of the work is: $Nm = J$ (joules), scalar physical quantity, it has only magnitude, no direction. By definition, in the general case, if the force $\vec{F}(\vec{r})$ is acting on the body at the point defined by the location vector $\vec{r}$, the body moves by $d\vec{r}$ and the elementary work is:

$$dW = \vec{F}(\vec{r}) \cdot d\vec{r} \,,$$

the total work between the points defined by the $\vec{r}_1$ and the $\vec{r}_2$ location vectors is:

$$\boxed{W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r}}\,.$$

The so-called linear integral of the displacement to force $\vec{F}$ is usually very difficult to calculate, but can be solved in special cases.

1.3.3. Examples for special works

1.3.3.1. The work of gravity and potential energy

On the surface of the Earth, and not too large in space and height, the body of mass m is affected by $m\vec{g}$ gravity. Calculate how much gravity works when the body moves from $\vec{r}_1$ to $\vec{r}_2$ along any curve.

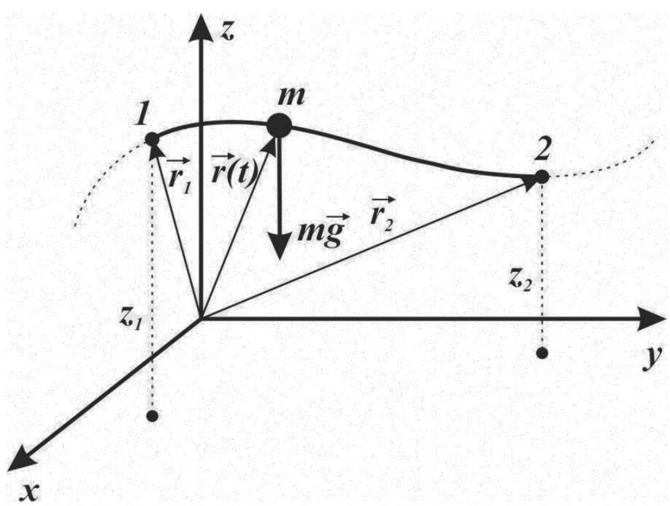

Figure 1.32

If the x-y plane is on the earth's surface then $\vec{F}(F_x; F_y; F_z) = \vec{F}(0;0;-mg)$ The coordinate of the location vectors $\vec{r}_1(x_1; y_1; z_1)$ and $\vec{r}_2(x_2; y_2; z_2)$. In case of elementary $d\vec{r}$ displacement the elementary work is:

$$dW = \vec{F}(\vec{r}) \cdot d\vec{r} = F_x dx + F_y dy + F_z dz = -mgdz$$

And the work of gravity between z_1 and z_2 heights:

$$W_{gravity} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r} = \int_{z_1}^{z_2} -mgdz = -mg[z]_{z_1}^{z_2} = mg(z_1 - z_2) = mgh$$

In the case of the body moving towards the Earth ($z_1 > z_2$), W is positive, and gravity will do the work. If the body is moved in the opposite direction to the gravitational field by a lifting force of $F_l = mg$ to h, the work of gravity W is negative and the work of lifting force $W = mgh$ is positive.

According to the work theorem, this is equal to the variation of the body energy ΔE. Let $h = 0$ be the height on the Earth's surface. Then:

$$\boxed{E_p = mgh}$$

quantity is called the potential (positional, altitude) energy related to the gravity and the position of the body above the surface of the earth. Its value is zero at the surface of the earth and negative under the surface of the earth.

Comment:

The above statement is not so strict. Since only the change of energy plays a role in physical processes, not the energy itself, height can be measured from anywhere.

1.3.3.2. The work of accelerating force and kinetic energy

Force is needed to accelerate bodies. Calculate the work done on the mass m accelerated by the constant magnitude and direction force F when accelerating it from a stationary position to a velocity v on a horizontal, frictionless line.

Based on the general definition of work:

$$W_{acc} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r} = Fs = ma\frac{a}{2}t^2 = \frac{1}{2}mv^2$$

According to the work theorem, this is equal to the change in body energy ΔE, which is due to the zero initial velocity, the energy associated with the motion of a mass m, the so-called. kinetic energy.

$$\boxed{E_k = \frac{1}{2}mv^2}$$

1.3.3.3. The work of spring force

Calculate the tensile force $\vec{F}_t = D\vec{x}$ that stretches the spring slowly (at a constant speed) against the spring force $\vec{F}_{sp} = -D\vec{x}$ and the elastic force exerted by a spring that extends from x_1 to x_2 (Figure 1.33).

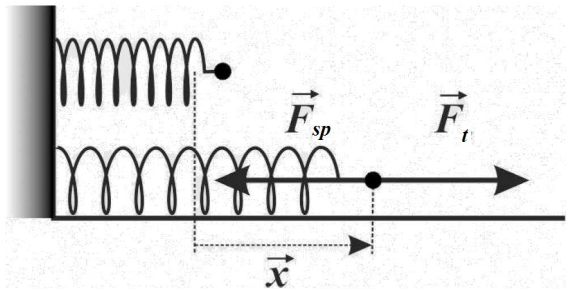

Figure 1.33

Based on the general definition of work:

$$W_t = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r} = \int_{x_1}^{x_2} Dxdx = D\left[\frac{x^2}{2}\right]_{x_1}^{x_2} = \frac{1}{2}D\left(x_2^2 - x_1^2\right)$$

According to the work theorem, this equals the change in the body energy ΔE, which is equal to the elastic energy stored in the spring of elongation at x considering the initial elongation $x_1 = 0$:

$$\boxed{E_e = \frac{1}{2}Dx^2}$$

At the same time, the work of the spring force F_{sp} opposite to the tensile force F_t:

$$W_{sp} = -W_{tens} = -\frac{1}{2}D\left(x_2^2 - x_1^2\right)$$

If $x_2 > x_1$ (the spring is stretched), W_t is positive (the tensile force is working), and W_{sp} is negative (working against the spring force).

If $x_1 > x_2$ (the spring is contracted), W_t is negative (spring force is working against tensile force) and W_{sp} is positive (spring force is working).

1.3.4. The performance

Performance numerically denotes the work done during a unit of time, denoted by P. If we perform W work during time t, then

$$\langle P \rangle = \frac{W}{t}$$

quotient gives the average performance. Its SI unit is $\frac{J}{s} = W$ (watt). According to the $W = Pt$ product, the most commonly used unit of work is Wh (1 Wh = $3.6 \cdot 10^3$ J) and its multiples (kWh, MWh).

If work is not performed evenly over time, the elementary performance is:

$$\boxed{P(t) = \lim_{\Delta t \to 0} \frac{\Delta W(t)}{\Delta t} = \frac{dW(t)}{dt} = \dot{W}(t)}.$$

if $dW = \vec{F} \cdot d\vec{r}$ then

$$P(t) = \vec{F} \cdot \frac{d\vec{r}(t)}{dt} = \vec{F} \cdot \vec{v}(t)$$

Performance, like work, is a scalar quantity.

1.3.5. Efficiency

In the interaction of bodies, only a fraction of the total work done is utilized. The remainder, which is infertile for work, is irreversibly used for other energy transfers, eg. friction generates heat, radiates, etc. The ratio of the useful work done in the interaction W_u and the total work invested by W_{tot} is the efficiency, denoted by η. Calculation:

$$\boxed{\eta = \frac{W_u}{W_{tot}}},$$

which is a ratio without a unit of measure, and its numerical value shows the amount of the fraction of work invested is considered useful. Because of the linear proportion between work and performance:

$$\eta = \frac{P_u}{P_{tot}}$$

According to the law of energy conservation, the useful work cannot be greater than the total work, so theoretically $\eta \leq 1$. In the case of $\eta = 1$, the total work would be fully utilized. Such a machine is called perpetuum mobile because the utilized energy could once again be fully recovered and invested, which would be fully utilized again, and so on. However, according to Thermodinamics, perpetuum mobile does not exist (see below), so practically always: $\eta < 1$ (100%).

2. Thermodynamics

In the previous section, some chapters of Classical Mechanics have been outlined. This discipline achieved such success in the 17th century; they sought to trace other physical phenomena and processes, including thermo-phenomena, to mechanical phenomena. The difficulty is that a large number of particles (atom, molecule, ion) are involved in thermal phenomena, and it is impossible to write an equation of motion for them individually. For example, it is not possible to determine (measure) the initial conditions for particles in the air present in the room, to write and solve the equations of motion in a "finite time", but in principle it is possible.

Thermal science is a discipline of physics that has developed its own research method and mathematical language. The test method can be divided into thermodynamics and molecular physics.

Thermodynamics is based on experience, relies on facts and measurable physical quantities, and tries to formulate general laws and describe them in mathematical languages.

Molecular physics interprets thermal phenomena based on material structure knowledge.

Kinetic gas theory interprets thermo concepts and thermal processes based on model-based assumptions about the mechanical movement of molecules. The theory takes into account that, because of the effect of a large number of particles, the interpretation can only be statistical. The authors of this were mainly James Clerk MAXWELL, Ludwig Eduard BOLTZMANN and Josiah Willard GIBBS in second half of the 19^{th} and beginning of the 20^{th} centuries. Max PLANCK is also associated with a refinement of theory towards quantum mechanics. The entropy introduced by Rudolf CLAUSIUS has also gained application in Information Theory and Cosmology.

2.1. Temperature and thermal expansion

2.1.1. The temperature

The concept of temperature is derived from the subjective perception of warm, cold, lukewarm words, and the physical quantity expressing the thermal state of bodies. This physical quantity cannot be derived from the three basic quantities (length, weight, time) used in the previous section, so it is not a derived quantity.

Three experiences, independent of our senses, allow us to interpret and measure temperature:

1. When two materials in different thermal conditions come into direct contact with each other, the warmer cools down, the cooler gets warmer until the thermal equilibrium, the common temperature, is reached. This is also called the 0^{th} law of thermodynamics.

2. As the temperature changes, most quantities of the physical properties of the materials change (eg volume, pressure, electrical resistance, etc.)

3. It is possible to obtain reproducible (reproducible) thermal conditions, temperatures, such as the melting point of ice at a given pressure or the temperature of the steam of boiling water.

Based on these, temperature measuring instruments and empirical temperature scales can be constructed. On the other hand, the so-called thermodynamic temperature scale (formerly known as the absolute temperature scale) is independent of the quality of the material. It is based on the temperature at which the thermal motion of the molecules ceases in principle. In practice, we use three temperature scales (Figure 2.1): Celsius, Fahrenheit and Kelvin.

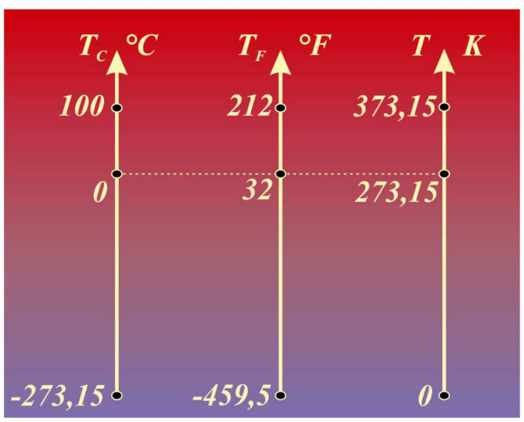

Figure 2.1

The relationships between them are:

$$T_F = \left(\frac{9}{5}T_C + 32\right)°F\,,\; T_C = \frac{5}{9}(T_F - 32)°C\,,\; T = (T_C + 273.15)K$$

2.1.2. Thermal expansion of homogeneous and isotropic solid bodies

Solid bodies are isotropic when their physical properties are the same in any direction. Linear, surface and volumetric (cubic) thermal expansion are distinguished. Thermal expansion is called linear if the cross-section of the body is negligible in relation to its length. Experience has shown that the change in body length Δl *is* proportional to the original length and temperature change:

$$\Delta l \sim l_0 \Delta T$$

The linear expansion law expressing this:

$$\Delta l = \alpha l_0 \Delta T \,,$$

where l_0 is the length corresponding to the initial temperature T_0 and α is the linear coefficient of thermal expansion of the material quality of the body, the unit of measurement being: $\frac{1}{K}$.

Because the temperature change is equal on the Celsius and Kelvin temperature scales, even the $\frac{1}{°C}$ unit can be used. Its magnitude in the case of metals is usually very small (in the order of 10^{-4}, 10^{-5}).

Taking into account that $\Delta l = l - l_0$ the linear expansion law can also be written in the form:

$$\boxed{l_T = l_0(1 + \alpha \Delta T) = l_0[1 + \alpha(T - T_0)]}.$$

which is mathematically a linear equation on the l–T diagram (Figure 2.2):

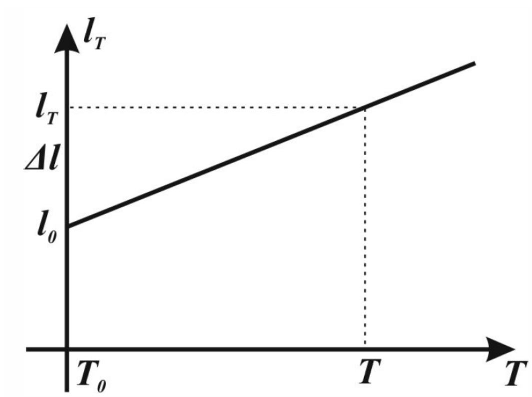

Figure 2.2

Comment:

α is generally considered to be constant only at temperature intervals.

2.1.3. Surface thermal expansion of solid bodies

If the area of the rectangle by initial temperature T_0 with initial side lengths a_0 and b_0 is $A_0 = a_0 b_0$, and at temperature T is $A_T = a_T b_T$ (Figure 2.3), then, using the law of linear expansion, the law of surface expansion is:

$$A_T = a_T b_T = a_0(1+\alpha\Delta T) b_0(1+\alpha\Delta T) = a_0 b_0 (1+\alpha\Delta T)^2 = a_0 b_0 (1 + 2\alpha\Delta T + \alpha^2 \Delta T^2).$$

Since α is very small, the member containing α^2 contributes only marginally to the result. By omitting this member, the surface thermal expansion law is:

$$\boxed{A_T = A_0(1+2\alpha\Delta T)}.$$

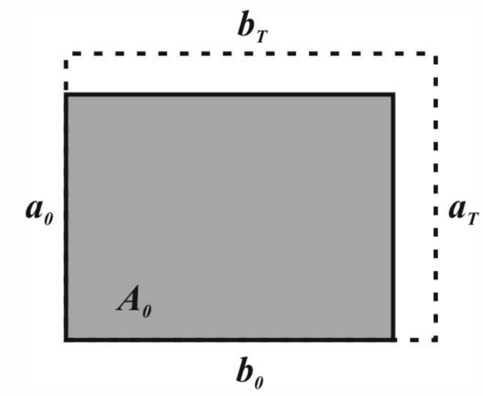

Figure 2.3

2.1.4. Volumetric (cubic) expansion of solid bodies and liquids

Following the example above for a cuboid $V_0 = a_0 b_0 c_0$ and therefore:

$$V_T = a_T b_T c_T = a_0 b_0 c_0 (1+\alpha\Delta T)^3 \approx V_0(1+3\alpha\Delta T)$$

where $3\alpha = \alpha_V$ (formerly β) is the volumetric (cubic) coefficient of thermal expansion.

The volume expansion of liquids is similar in volume to the relation to the volume expansion of solids:

$$\boxed{V_T = V_0(1+\alpha_V\Delta T)}$$

where α_V is the volumetric thermal expansion coefficient of the liquid.

It is often the case that the density of the large solid body, but more particularly the fluid, is affected by the change in temperature. Dividing mass of expanding material (unchanged) by $V_T = V_0(1+\alpha_V\Delta T)$ relation:

$$\frac{m}{V_T} = \frac{m}{V_0(1+\alpha_V\Delta T)},$$

of which the density at temperature T:

$$\rho_T = \frac{\rho_0}{1 + \alpha_V \Delta T}$$

If $\alpha_V \Delta T \ll 1$, then using the $\frac{1}{1+x} \approx 1 - x$ mathematical approximation, the density at T is:

$$\boxed{\rho_T \approx \rho_0 (1 - \alpha_V \Delta T)}$$

arises. Water behaves in a peculiar way, as it shrinks from 0ºC to 4ºC on heating and expands above 4ºC. Therefore, the water density has a maximum at 4ºC (Figure 2.4).

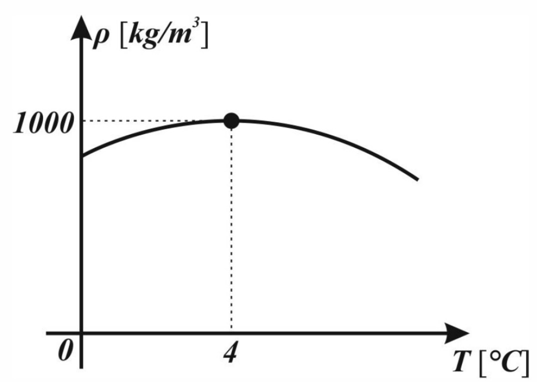

Figure 2.4

2.2. The thermal state equation of ideal gases

A gas is called ideal when the particles collide flexibly with one another and with the vessel wall, but do not exert any attractive or repulsive force, and make a smooth linear motion between the two collisions, with a negligibly small volume compared to the vessel volume.

Quantities characterizing the ideal gas state of a given quantity and quality are the thermal status indicators. The status indicators are pressure p, volume V, temperature T, and material quantity n. The latter is the quotient of the mass m of the gas and the molar mass M of the gas (SI unit: $\frac{kg}{mol}$), i.e. $n = \frac{m}{M}$. The function relation between these is the state function described by the empirical thermal state equation:

$$\boxed{pV = nRT},$$

where T is the thermodynamic temperature in Kelvin and R is the molar gas constant (formerly known as the universal gas constant).

According to the Avogadro Law:

"The same volume of different gases contains the same number of particles at the same pressure and temperature."

The molar gas constant R can be calculated from the experience that the volume of the ideal gas with $n = 1$ mol, $p = p_0 = 101325\ Pa$ and temperature $T = T_0 = 273.15\ K$, ie normal state gas, is $V = V_0 = 22.41 \cdot 10^{-13}\ m^3$ (standard molar volume). Then, based on the thermal state equation:

$$R = \frac{p_0 V_0}{n T_0} = \frac{101325\ Pa \cdot 22.41 \cdot 10^{-13}\ m^3}{1\ mol \cdot 273.15\ K} = 8.31 \frac{J}{mol \cdot K}$$

Often, we also use the quotient of molar gas constant R and the Avogadro constant N_A ($= 6.02 \cdot 10^{23} \frac{1}{mol}$), the

$$k = \frac{R}{N_A} = \frac{8.31 \frac{J}{mol \cdot K}}{6.02 \cdot 10^{23} \frac{1}{mol}} = 1.38 \cdot 10^{-23} \frac{J}{K}$$

Boltzmann constant. By introducing this, the thermal state equation of the ideal gases is

$$pV = nRT = nkN_A kT = NkT$$

where N is the number of particles of the gas.

2.3. Changes in the state of the ideal gases

When comparing the states of a given quantity of an ideal gas at two different thermal equilibrium states (p_1, V_1, T_1, n and p_2, V_2, T_2, n), applying the thermal state equation to both states:

$$p_1 V_1 = nRT_1$$

and

$$p_2 V_2 = nRT_2 .$$

The quotient of these is

$$\frac{p_1 V_1}{p_2 V_2} = \frac{nRT_1}{nRT_2},$$

thus

$$\boxed{\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}}$$

is the so-called combined gas law, which expresses the relationship between the two thermal equilibrium states of the same ideal gas. The advantage is that neither the molar gas constant R nor the Boltzmann constant k is included.

Depending on which state determinant does not change with n = constant, we distinguish three special, empirically verifiable state changes: isobaric, isochoric and isothermal.

2.3.1. Isobaric state change

The state change of the ideal gas is isobaric when its pressure does not change during the process ($p_1 = p_2$). If the pressure p of the ideal gas and the quantity of matter n are constant, then in thermal equilibrium from the combined gas law:

$$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$$

This law is called Gay-Lussac's 1st Law.

The isobaric change in the p–V diagram, V–T diagram and p–T diagram is shown in Figure 2.5.

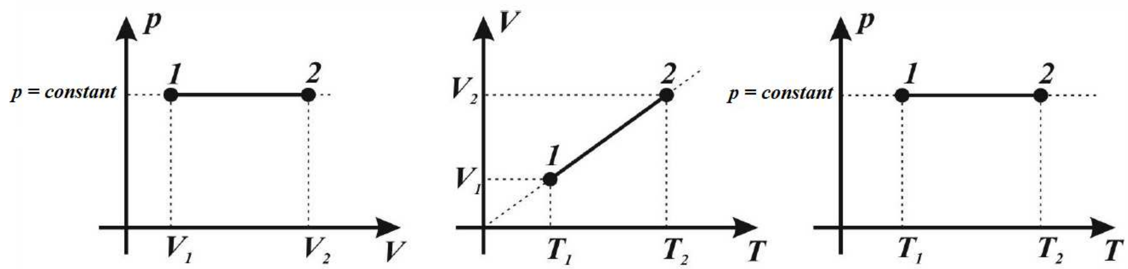

Figure 2.5

Calculate the work performed by the gas column which has an isobaric change of state (p = constant) from T_1 to T_2 in the cylinder with initial length h closed by the piston with surface area A.

According to Gay-Lussac's 1st law, the change in the ideal gas volume is:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

from where

$$V_2 = V_1 \frac{T_2}{T_1}$$

and

$$\Delta V = V_2 - V_1 = V_1 \left(\frac{T_2}{T_1} - 1 \right).$$

The ideal gas pushes the piston outward on s path length with constant force $F = pA$, while its volume increases from V_1 to V_2. Its volumetric work is:

$$W = -Fs = -pAs = -p\Delta V = -pV_1\left(\frac{T_2}{T_1} - 1\right),$$

which by convention is a negative value when the gas is working on the piston (dilates), or positive when an external force is working on the gas closed off by the piston (compressed) and illustrated for the initial and final states 1 and 2 in the pV diagram is equal to the area below the line (isobar) (Figure 2.6).

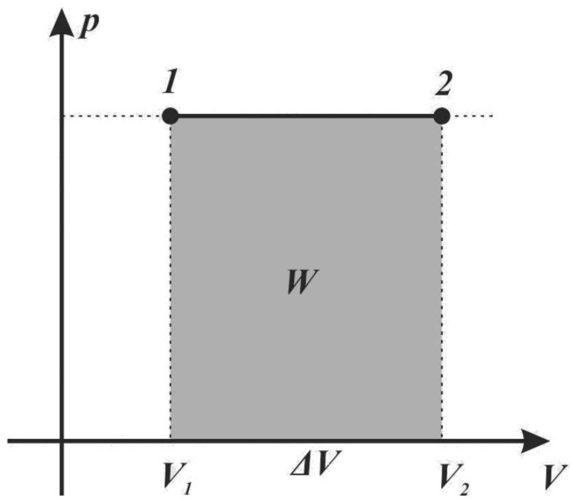

Figure 2.6

It is generally true that the W work performed by the ideal gas in its state change is numerically equal to the area under the arc defined by the initial 1 and end 2 states of the change in the p–V diagram.

2.3.2. Isochoric state change

The state change of the ideal gas is isochoric when its volume does not change during the process ($V_1 = V_2$). If the volume V of the ideal gas and the quantity of matter n are constant, then in thermal equilibrium from the combined gas law:

$$\boxed{\frac{p_1}{T_1} = \frac{p_2}{T_2}}$$

This law is called Gay-Lussac's 2nd Law.

The isochoric change in the p–V diagram, V–T diagram and p–T diagram is shown in Figure 2.7.

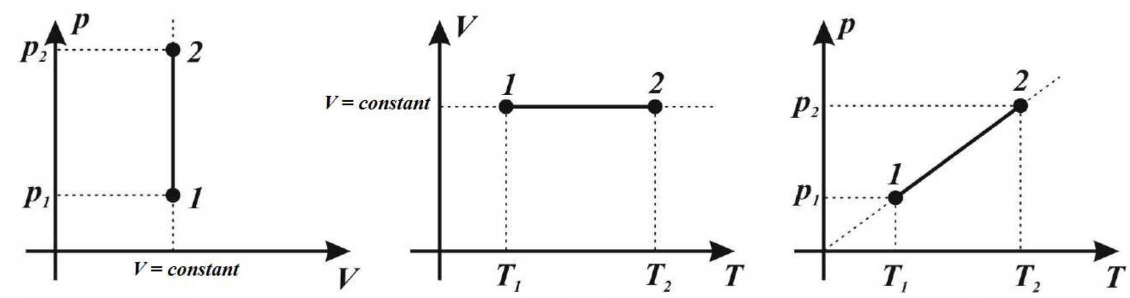

Figure 2.7

In the isochor state change, V = constant, $\Delta V = 0$, and hence the volumetric work W is zero:

$$W = 0.$$

2.3.2. Isothermal state change

The state change of the ideal gas is isothermal when its temperature does not change during the process ($T_1 = T_2$). If the temperature T of the ideal gas and the quantity of matter n are constant, then in thermal equilibrium from the combined gas law:

$$\boxed{p_1 V_1 = p_2 V_2}$$

This is the Boyle-Mariotte Law.

The isothermal change in the p–V diagram, V–T diagram and p–T diagram is shown in Figure 2.8.

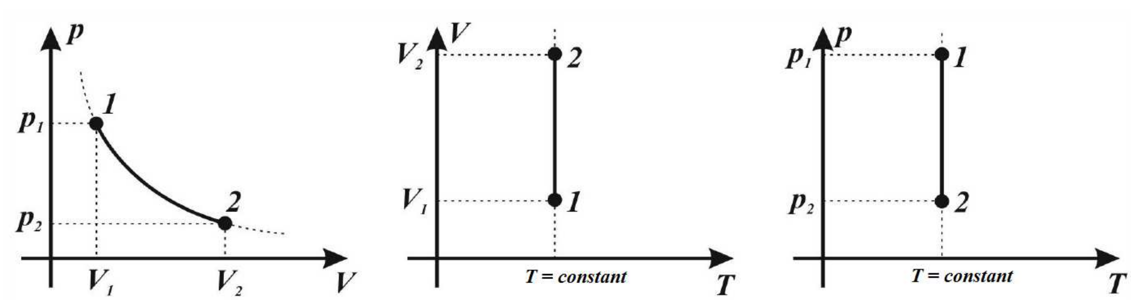

Figure 2.8

Calculate the work performed by the gas at constant temperature T from volume V_1 to volume V_2.

We have seen that the work of the expanding gas W is given by the area under the arc at the beginning and end of the curve representing the change of state in the p–V diagram (Figure 2.8). Its value is negative at gas expansion, so:

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = -\int_{V_1}^{V_2} p(V) dV .$$

Using the $pV = nRT$ state equation, the volumetric work is:

$$W = -\int_{V_1}^{V_2} p(V)dV = -\int_{V_1}^{V_2} \frac{nRT}{V} dV .$$

At constant T temperature after integration:

$$W = -nRT[\ln V]_{V_1}^{V_2} = -nRT \ln \frac{V_2}{V_1} ,$$

or using the Boyle-Mariotte law $p_1V_1 = p_2V_2$:

$$W = -nRT \ln \frac{p_1}{p_2}$$

(see Figure 2.9).

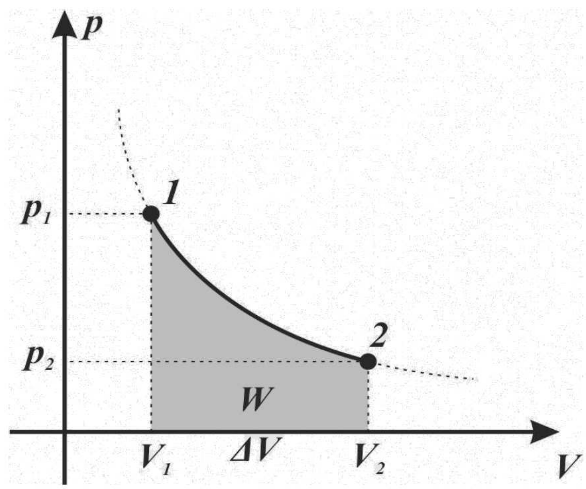

Figure 2.9

2.4. Energy characteristics of ideal gases

2.4.1. Internal energy

Experience has shown that the ideal gas in the piston cylinder expands when heated, pushes the frictionless piston and works against the outside environment. However, the expanding gas can only work if it has the ability to work (energy).

The energy E of any macroscopic material of mass m (directly perceptible to or perceived by the naked eye or other senses) is composed of two parts. The sum of kinetic energy $E_k = \frac{1}{2}mv^2$ and potential energy $E_p = mgh$, ie the macroscopic energy $E_m = \frac{1}{2}mv^2 + mgh$ and the U internal energy:

$$\boxed{E = E_m + U}.$$

In thermology, internal energy is defined as the sum of the translational, rotational and vibration energy of the system particles in proportion to the thermodynamic temperature T. The number of independent movements of the particles is the so-called. degree of freedom (see below).

The internal energy of an ideal gas of material n, degree of freedom f, at thermodynamic temperature T is:

$$\boxed{U = \frac{f}{2} nRT}$$

or using the thermal state equation

$$U = \frac{f}{2} NkT = \frac{f}{2} pV \ .$$

An important property of internal energy is that its change is clearly defined by the initial and final states of the ideal gas (p, V, T and n). Thus the internal energy is being a state function.

Comment:

For a description of the calculation of internal energy, see chapter Elements of Molecular Physics.

2.4.2. Heat

The ΔU change in the internal energy U of the macroscopic ideal gas consisting of gas particles which is disordered by heat movement is directly proportional to the thermodynamic temperature change ΔT. This energy transfer associated with disorderly heat movement without work (and impulse transfer) is called heat transfer, and the internal energy transferred during heat transfer is referred to as heat, denoted by Q, SI joules (J), so:

$$\boxed{\Delta U = Q}.$$

The amount of heat is the amount of energy exchanged in a thermal way and can only be talked about during the thermal interaction. After the thermal interaction occurs, we can only talk about internal energy, the change in internal energy due to thermal interaction is the amount of heat.

2.4.3. Specific heat and heat capacity

Experience has shown that the amount of heat Q emitted or absorbed during thermal energy exchange is directly proportional to the body temperature ΔT:

$$Q = C\Delta T \ ,$$

where the proportionality coefficient C is the heat capacity of the specific amount of material, its SI unit is $\frac{J}{K}$ or $\frac{J}{°C}$. The heat capacity of a unit mass is called specific heat or specific heat capacity, denoted by c, that is:

$$c = \frac{C}{m},$$

its SI unit is $\frac{J}{kg \cdot K}$ or $\frac{J}{kg \cdot °C}$. Definition: $1\frac{J}{kg \cdot K}$ is the specific heat of a 1 kg mass if the temperature change of $\Delta T = 1\ K = 1\ °C$ requires $Q = 1\ J$. The specific heat of water is eg $4200\frac{J}{kg \cdot K}$.

In the case of gases, there are two types of specific heat, depending on whether the given gas is exchanging heat at constant volume (c_V) or constant pressure (c_p). There is a $c_p > c_V$ relation and the ratio of the two:

$$\frac{c_p}{c_v} = \kappa,$$

where κ is the so-called. adiabatic constant, about 1.66 for one-atom gases (such as helium and argon), about 1.4 for the two-atom molecule (such as nitrogen, oxygen, and hydrogen), about 1.3 for the three or more atomic gases (such as carbon dioxide and acetylene).

The Robert Mayer equation also establishes a relationship between the specific heat of ideal gases at constant pressure c_p and constant volume c_V:

$$c_p - c_v = \frac{R}{M},$$

where M is the molar mass of the gas.

In addition to the known state changes, we also mention the adiabatic state change. An adiabatic state change is the state of a gas when there is no heat exchange between the gas and its surroundings, i.e., $\Delta Q = 0$ (e.g., the gas is compressed rapidly so that there is no time for heat exchange, i.e. temperature equilibration). In this case

$$pV^{\kappa} = constant,$$

that is, applied to two equilibrium states:

$$p_1V_1^{\kappa} = p_2V_2^{\kappa}.$$

The adiabatic state change is illustrated in the p–V diagram by an adiabatic curve. Since $\kappa > 1$, the adiabata is steeper than the isotherm (Figure 2.10):

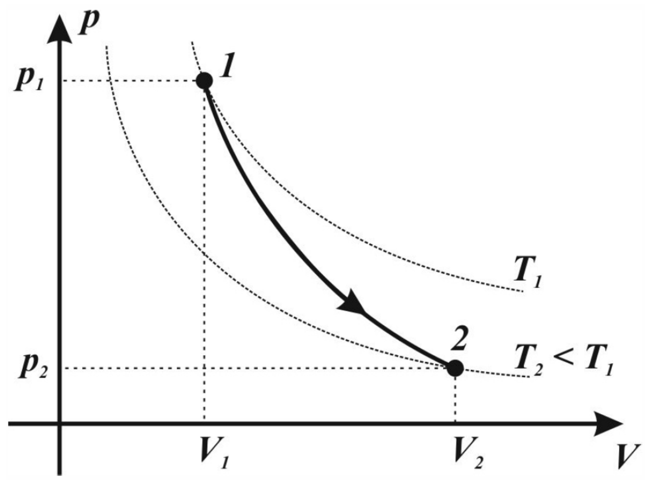

Figure 2.10

2.5. State changes and phase transitions

The state of the bodies changes due to heat transfer (heat dissipation). This is how we talk about melting (reverse freezing) and evaporation (reverse condensation). The evaporation of solids is called sublimation (reverse crystallization).

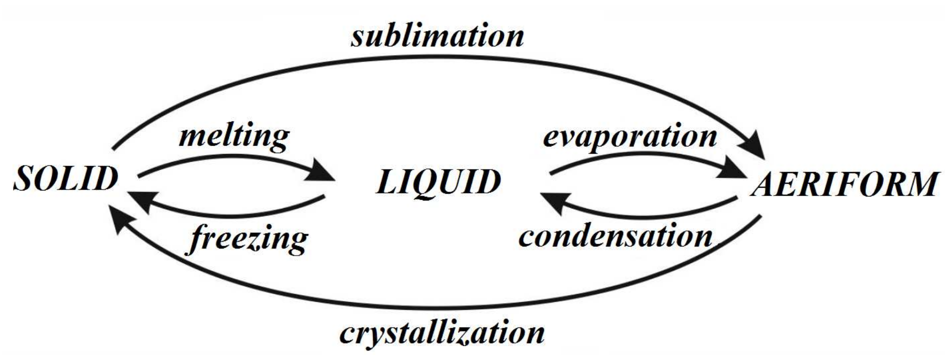

Figure 2.11

By constantly changing the temperature of a substance, if at a well-defined temperature a sudden change occurs in a physical property of the substance (eg. volume change), the temperature is called phase transition temperature. The simplest phase transition is characterized by the fact that at the conversion temperature the volume V of the material and the internal energy of U change in leaps and bounds either by heat absorption (eg by melting) or by heat transmission (eg by freezing). Such a transformation is called a state change when the material involved in the process is one-component, that is, it is made up of the same atoms (eg copper, aluminium) or the same molecules (eg water, carbon dioxide). Changes in the physical state may include melting, freezing, vapor formation (evaporation, boiling), condensation, sublimation, and crystallization (Figure 2.11).

The quotient $L = \frac{Q}{m}$ of the Q required heat for complete transformation and the m mass of the substance at the normal transformation temperature (normal melting point, normal freezing point, normal boiling point) by normal atmospheric pressure ($p = 101325\ Pa$) is called the specific transformation heat. SI unit: $\frac{J}{kg}$. The numerical value shows how much heat Q must be administered (endothermic process) or subtracted (exothermic process) for the phase change of a unit mass. Specific transformation heats for different state changes:

- specific melting heat
- specific freezing heat
- specific heat of evaporation
- specific boiling heat
- specific heat of condensation
- specific sublimation heat
- specific heat of crystallization

When heat is applied to any given material, its temperature rises, but during the transformation its temperature does not change until the material is completely transformed (Figure 2.12):

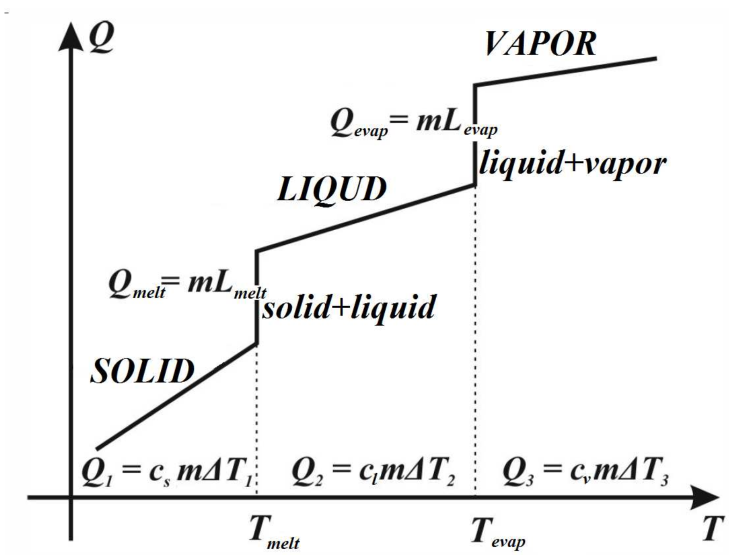

Figure 2.12

During melting and freezing, the heat changes the internal energy without changing the temperature and the mechanical work is negligible. However, in the case of evaporation and boiling, work on external air pressure is already considerable (the volume change is not negligible).

Chemically homogeneous ("pure material"), that is, in a one-component system, one or more phases are possible simultaneously. At a given temperature and pressure, two phases can be in equilibrium. By this also means that in a closed volume, the weight ratio of the two phases is unchanged over time, that is, a dynamic equilibrium, i.e., the same number of particles passes from one phase to another in the same time. The number of biphasic states is three; vapor–liquid, vapor–solid and liquid–solid.

In phase equilibrium, the so-called phase-curves connecting the related pressure-temperature points are usually illustrated on the p–T phase diagram. The phase curves connecting the different p–T phase pairs intersect at one point, the so-called H triple point (Figure 2.13):

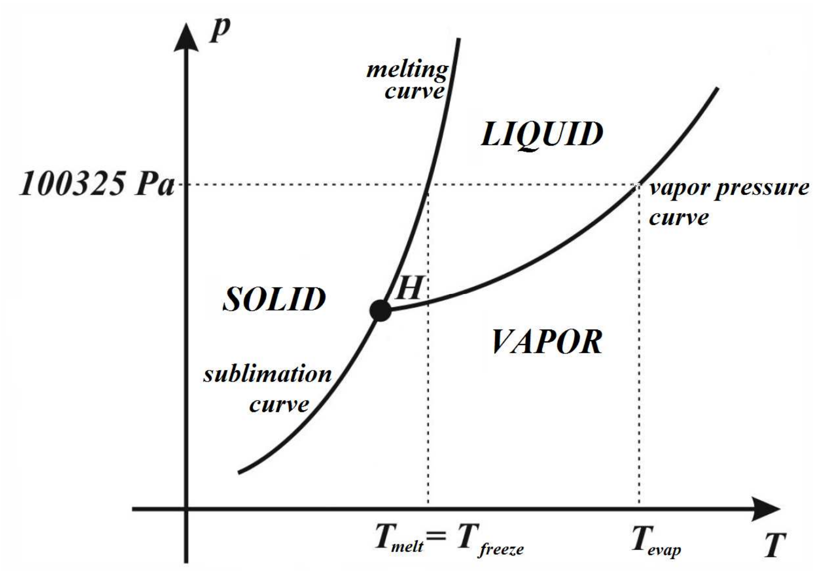

Figure 2.13

Note that the triple point temperature of the ice-water-steam system 0.01 °C = 273.16 K is not the normal freezing point of water 0 °C = 273.15 K at which ice and water are in equilibrium at normal atmospheric pressure (Figure 2.14):

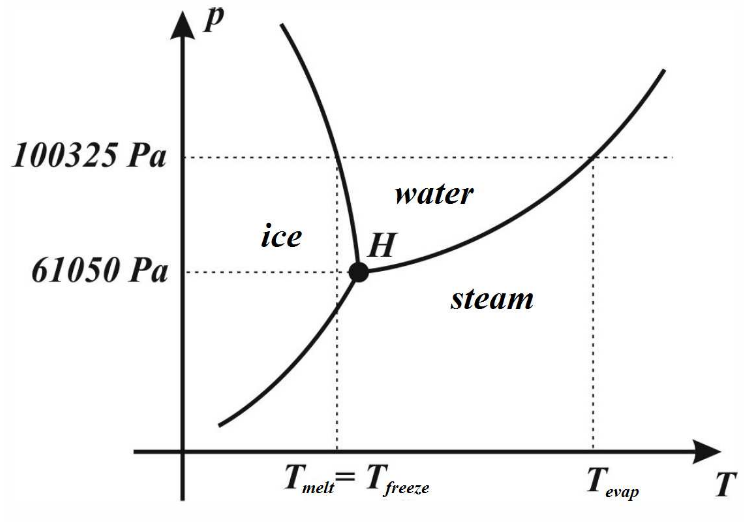

Figure 2.14

2.6. Principles of thermodynamics

2.6.1. The 1st principle of thermodynamics

The first principle of thermodynamics is the extension of the principle of energy conservation to thermodynamic processes. It states that:

“The internal energy of the bodies can be changed by thermal (heat exchange) and / or mechanical work.”

So the equation for energy conservation:

$$\boxed{\Delta U = Q + W}.$$

Comment:

- *Q is positive if the body (system) absorbs heat from its surroundings.*
- *Q is negative if the system releases heat to its surroundings*
- *W is positive if the environment is working against the system*
- *W negative if the system is working against the environment*
- *ΔU is positive if the internal energy of the system increases during the process*
- *ΔU is negative if the internal energy of the system decreases during the process*

If the system only performs volumetric work, then the system performs volumetric work is:

$$W = -\int_{V_1}^{V_2} p(V)dV$$

so the 1st principle is:

$$\boxed{\Delta U = Q - \int_{V_1}^{V_2} p(V)dV}.$$

At constant volume, $W = 0$, so $\Delta U = Q$, ie the change in the internal energy of the system is equal to the heat transmitted or absorbed.

1st principle is not derived from the laws of experience, so it is a fundamental law of nature. 1st principle is also usually written as:

"There is no periodic thermodynamic machine ($\Delta U = 0$), the so-called first-class perpetual mobile, which is capable of working without heat input."

2.6.2. The 2nd principle of thermodynamics

We have seen that processes involving heat absorption or heat dissipation take place in accordance with the law of energy conservation. However, the 1st principle does not account for the direction of natural processes over time. For example, the kinetic energy of a falling body, when impacted, converts to heat in the falling body and its surroundings. However, the opposite is not the case: at the expense of the internal energy of the body and the earth, the body does not rise, although this would be permissible under 1st principle. Accordingly, the 2nd principle of thermodynamics states that:

“Real processes that occur during thermal interaction are always irreversible.”

Several equivalent terms are known for the 2nd principle in terms of physical content:

“The heat cannot itself be transferred from a lower temperature to a higher temperature. "

"It is not possible to make a periodically operating thermal engine which, by removing heat from a heat tank (eg seawater, air, soil), is capable of converting it completely into mechanical work."

"No second-class perpetual mobile can be created that can continuously transform the invested heat to work with circular processes."

The 2nd principle defines the direction in which a process takes place on its own. Namely, only processes can take place by themselves, where the intense state indicators (*p* and *T*) that are independent of the mass and volume of the system, are brought closer to equilibrium. That is, these processes result in the equilibrium of the intense state indicators of the system.

The amount of disorder (motion, energy distribution) of particles in a given material system (eg gas) is the entropy introduced by Rudolf Julius Emanuel CLAUSIUS, which, like U internal energy, is a state function, denoted by *S*. This change between initial state 1 and end state 2 is defined as:

$$\Delta S = \frac{\Delta Q}{T} = \int_1^2 \frac{dQ}{T}.$$

Its value describes the possible direction of the thermal processes. The SI unit of entropy is $\frac{J}{K}$.

Positive entropy change means the process of equalizing the intense state indicators of the system, in which case the disorder of the particles that make up the system increases. The 2nd principle of thermodynamics is therefore:

"The entropy of a closed system is always increasing, at equilibrium it reaches maximum."

That is, mathematically:

$$\boxed{\Delta S \geq 0}.$$

Experience has shown that in real processes, a certain amount of work or heat is always wasted. As a result, the efficiency of irreversible circular processes:

$$\eta = \frac{W_u}{W_{tot}} = \frac{|W|_u}{Q_{acquierd}} = \frac{Q_{acquierd} - |Q|_{transmitted}}{Q_{acquierd}} < 1$$

The 2nd principle of thermodynamics describes the operation of heat engines, refrigerators and heat pumps.

Thermal power plants convert thermal energy into mechanical energy and can be of a steam engine (piston steam engine, steam turbine) or gas engine (gas turbine, internal combustion engine: gasoline and diesel). During their operation, they take $Q_{acquired}$ heat from the heat source and release $Q_{transmitted}$ heat to their surroundings while performing $|W|_u = Q_{acquierd} - |Q|_{transmitted}$ useful work.

Refrigerators remove heat from their interior and, by external (usually electrical) work, transfer heat to a higher temperature environment.

Heat pumps are heaters based on the principle of refrigerators. In winter, the building is heated by removing heat from the colder heat tank (soil, ponds, rivers) and supplying heat to the higher temperature building.

2.6.3. The 3rd principle of thermodynamics

The 3rd principle of thermodynamics states that:

Towards 0 K, the entropy of chemically homogeneous solids and liquids is zero, ie:

$$\boxed{\lim_{T \to 0} S = 0}.$$

Also, at $T \to 0$ K, the specific heat approaches zero and consequently a small amount of heat causes a considerable temperature change. The 3rd principle of thermodynamics in other words is therefore:

It is impossible to create a machine that works periodically, so called third-class perpetual mobile capable of producing a temperature of 0 K.

And since the bodies are not completely insulated from their surroundings, they cannot be cooled to $0\,K$. However, $0\,K$ can be closely approached: minimum temperature created in laboratory was $4.5 \cdot 10^{-10} K$.

2.7. Elements of molecular physics

As we have already pointed out, it is practically impossible to describe the movement of a large number (approx. 10^{24} pieces) of macroscopic materials (bodies) by classical mechanics. The combined effect of many particles determines the macroscopic features that are already observable and measurable. Therefore, the system is characterized by the average particle movement rather than by tracking the movement of each particle accurately. In addition to the familiar laws of physics, a new approach, mathematical statistics, and probability theory are also needed.

2.7.1. The pressure and temperature of the ideal gas

Experience has shown that the physical properties of different gases are very similar. This makes it possible to interpret the properties of gases in a single structural model, the ideal gas model.

The basic assumptions of gas theory are as follows:

1. Gas particles (atoms, molecules) are small spheres that move irregularly and collide elastically with each other and the vessel wall.
2. There is no interaction other than a flexible collision interaction.
3. The total volume of the gas particles is negligible compared to the volume occupied by the gas.

2.7.1.1. The pressure of the ideal gases

Let V be a rectangular vessel of ideal gas of mass m. Determine the pressure $p = \frac{F}{A}$ of the gas on the surface A of the vessel.

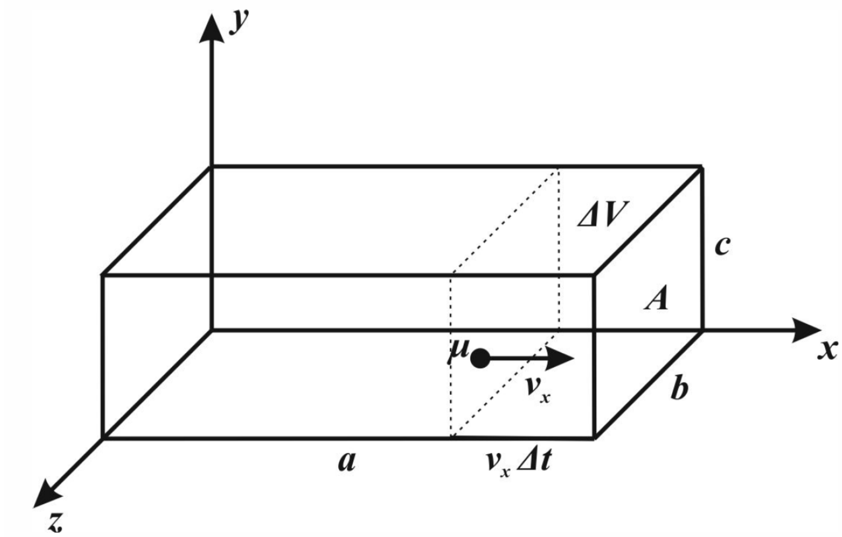

Figure 2.15

A vessel of volume $V = abc$ shall contain N moving particles of mass μ each. During time Δt, particles whose distance from the wall is $v_x \Delta t$ will collide with the side panel of the vessel (Fig. 2.15) thus are in the compartment

$$\Delta V = A v_x \Delta t .$$

If the concentration of the particles is $c = \dfrac{N}{V}$, then in the volume ΔV

$$\Delta N = c \Delta V$$

particles can be found. Half of these particles move in the positive direction of the x-axis (the other half in the opposite direction), that is:

$$\Delta N = \frac{1}{2} c \Delta V = \frac{1}{2} c A v_x \Delta t .$$

The ΔI change in impulse I of particle of mass μ and velocity v_x after elastic collision with wall is:

$$\Delta I = \mu v_x - \left(- \mu v_x\right) = 2 \mu v_x .$$

Therefore, the change in the x component of the impulse of each particle impinging on the surface A is:

$$\Delta I_x = \Delta N \Delta I = \frac{1}{2} c A v_x \Delta t 2 \mu v_x = c \mu A v_x^2 \Delta t .$$

Pressing force F applied to the wall by impinging particles over Δt time, using the above mentioned:

$$F_x = \frac{\Delta I_x}{\Delta t} = c\mu A v_x^2 ,$$

and the pressure p on the wall is:

$$p = \frac{F_x}{A} = c\mu v_x^2 .$$

Accurate calculation must take into account that the velocity of the gas particles is different (v_x is not constant), so instead of v_x^2 the average of the squares $\langle v_x^2 \rangle$ in the x direction must be taken into account, ie the pressure is:

$$p = c\mu \langle v_x^2 \rangle .$$

The consequence of random motion is that

$$\langle v_x^2 \rangle = \langle v_z^2 \rangle = \langle v_z^2 \rangle = \frac{\langle v^2 \rangle}{3} .$$

With this in mind

$$p = c\mu \frac{\langle v^2 \rangle}{3} ,$$

where the ideal gas pressure after expanding the quotient by 2:

$$p = c\mu \frac{\langle v^2 \rangle}{3} \cdot \frac{2}{2} = \frac{2}{3} \cdot \frac{N}{V} \cdot \left(\frac{1}{2} \mu \langle v^2 \rangle \right)$$

where $\langle \varepsilon_k \rangle = \frac{1}{2} \mu \langle v^2 \rangle$ it the average kinetic energy of a single particle.

After multiplying by V:

$$pV = \frac{2}{3} N \langle \varepsilon_k \rangle$$

which will be used in the next section.

2.7.1.2. The temperature of the ideal gases

If we compare the above result with the $pV = NkT$ of the thermal state equation of ideal gases, then

$$NkT = \frac{2}{3} N \langle \varepsilon_k \rangle$$

From this the temperature of ideal gases:

$$\boxed{T = \frac{2}{3k} \langle \varepsilon_k \rangle} .$$

The average kinetic energy

$$\langle \varepsilon_k \rangle = \frac{3}{2} kT$$

of a single particle is solely a function of temperature.

If the ideal gas consists of $N = nN_A$ particles, then the average total kinetic energy of the gas particles is:

$$\langle E_k \rangle = N \langle \varepsilon_k \rangle = N \frac{3}{2} kT = nN_A \frac{3}{2} kT = \frac{3}{2} nRT \ .$$

2.7.3. The equipartition theorem

The average kinetic energy $\langle \varepsilon_k \rangle$ of the one-atom gas (He, Ne, Ar, Kr, Xe, Rn) particle of mass μ considered to be point-like, moving at an average velocity $\langle v \rangle$, is as follows:

$$\langle \varepsilon_k \rangle = \frac{1}{2} \mu \langle v_x^2 \rangle + \frac{1}{2} \mu \langle v_y^2 \rangle + \frac{1}{2} \mu \langle v_z^2 \rangle = \frac{3}{2} kT = 3 \frac{1}{2} kT \ ,$$

where $\langle v_x^2 \rangle$, $\langle v_y^2 \rangle$ and $\langle v_z^2 \rangle$ are the average of the velocity component squares and their number in the case of atomic particles with only translational motion is 3. By convention, the number of independent square members in the energy expression is called the number of degrees of freedom of the particle. Thus, the atomic gas particles that are only have translational motion have a degree of freedom of $f = 3$, and thus their average motion energy is:

$$\langle E_k \rangle = N \langle \varepsilon_k \rangle = N 3 \frac{1}{2} kT \ .$$

The two-atom gas molecules are capable of translational motion in the x, y, and z directions, and in a two-way rotational motion perpendicular to their axis. These give $f = 3 + 2 = 5$ degrees of freedom, their average motion energy is:

$$\langle E_k \rangle = N 5 \frac{1}{2} kT \ .$$

Final conclusion:

"One particle has $\frac{1}{2} kT$ energy on 1 degree of freedom of and $\frac{f}{2} kT$ one f degree of freedom. The average energy $\langle E_k \rangle = Nf \frac{1}{2} kT$ of the N particle system is evenly distributed over each degree of freedom."

This theorem for the even distribution of energy, the so-called equipartition theorem was recognized by Maxwell and Boltzmann (1860).

The average energy $\langle E \rangle$ of the ideal gas consisting of N particles, depending on the temperature T related to the translational, rotational and vibrating motion of the particles, is called the internal energy U of the ideal gas:

$$\boxed{U = Nf\frac{1}{2}kT = f\frac{1}{2}nRT}.$$

2.7.4. Statistical interpretation of entropy

Determine the number of ways $N = 4$ theoretically distinct particles can be placed in two ($g = 2$) equal volumes (Figure 2.16).

The $N = 4$ particles in the two spaces can be arranged in five ways, we say that the system has 5 macro states (Table 2.1). To distinguish the particles, we designate the particles with the letters a, b, c, d. With this in mind, the so-called Y is the number of micro states, which shows how many micro states a given macro state can implement. The Y number of micro-states that show how many gas particles may be present in the right space, for example, can be calculated mathematically by the k-th combination of n elements (particles) on the right, for example:

$$C_n^k = \frac{n!}{k!(n-k)!} = \binom{n}{k},$$

read: n under k.

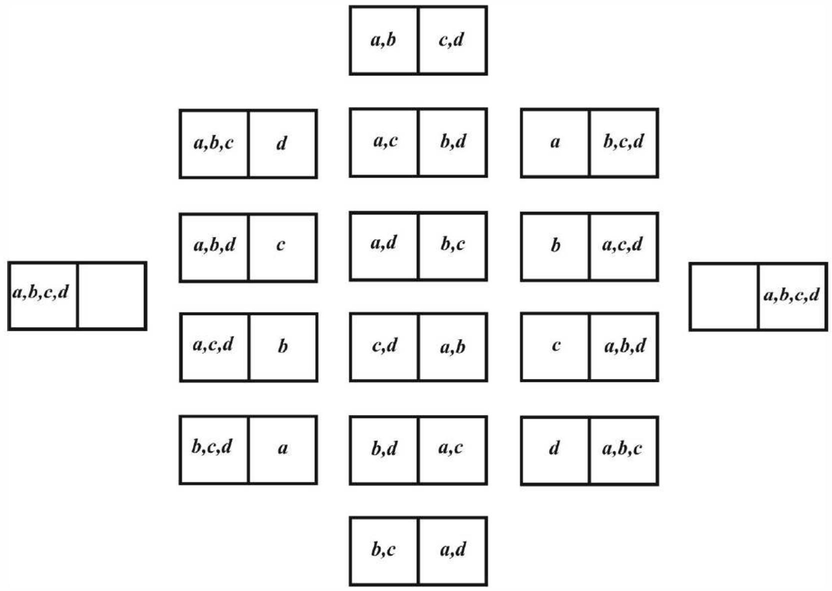

Figure 2.16

In our case, $n = N = 4$, $k = 0, 1, 2, 3, 4$ and the number of particles on the right side of each macro state, ie:

$$Y_N^k = \frac{N!}{k!(N-k)!} = \binom{N}{k}.$$

If we count the number of Y micro-states that a given macro-state can fulfil, then the mathematical probability P of that macro-state can be calculated: the ratio of the number of favorable cases Y to the total number of $N_{all} = g^N$ cases:

$$P = \frac{N_{favorable}}{N_{all}} = \frac{Y}{g^N}$$

Accordingly, the possible macro-states of the distribution of $N = 4$ distinct particles (a, b, c, and d) in two equal spaces ($g = 2$), the number of particles in each macro the number of micro-states Y and the mathematical probability P of a given macro state are shown in the following table:

Macro-states	number of particles in the right space k	micro-states $Y = \binom{4}{k}$	probability of the given macro state $P = \frac{Y}{g^N} = \frac{Y}{2^4} = \frac{Y}{16}$
1	4: *abcd*	1	$\frac{1}{16} = 0.0625 \to 6.25\%$
2	3: *abc, bcd, abd, acd*	4	$\frac{4}{16} = 0.25 \to 25\%$
3	2: ab, bc, cd, ad, bd, ac	6	$\frac{6}{16} = 0.375 \to 37.5\%$
4	1: a, b, c, d	4	$\frac{4}{16} = 0.25 \to 25\%$
5	0: –	1	$\frac{1}{16} = 0.0625 \to 6.25\%$
		Total: $16 = g^N = 2^4$	**100%**

Table 2.1

The number of micro-states belonging to one macro-state is called thermodynamic probability (statistical weight). If $N = N_1 + N_2 + \ldots + N_n$ particles are located in cell g db with N_i particles in cell i, then it can be shown that the number of micro states implementing a given macro state is:

$$Y = \frac{N!}{N_1!N_2!\ldots N_n!}.$$

For N particle number and g cell number, the total number of micro states in the system is g^N. The quotient of the number of micro states Y of a given macro state and the number of possible micro states of the system g^N is the so-called probability of finding:

$$P = \frac{Y}{g^N},$$

which shows the mathematical probability that the system is in a given macro state.

The thermodynamic and mathematical probabilities are not the same, but their ratio for two macro states is the same. The macro-state it most likely that can be accomplished by most of the micro-states (example 3). Gases fill their space evenly because most micro-states belong to it, and this is most likely to occur.

2.8. The spread of heat

In the chapters discussed so far, the experience of temperature equilibration between materials of different temperatures when they interact with one another played a crucial role. According to 2^{nd} principle of thermodynamics, the direction of temperature change is such that the heat always goes from a higher temperature to a lower temperature.

At this point, let's examine how heat can be transferred from one place to another. There are basically three forms of this; heat conduction, heat flow and heat radiation.

2.8.1. The heat conduction

Heat conduction occurs when higher velocity particles transfer part of their kinetic energy to lower velocity particles during collisions resulting from disordered thermal motion. There is no particle flow here, the heat conduction is characteristic of solid bodies. Specifically in metals, thermal conductivity (and electrical conductivity). is related to orderly movement of the so-called free electrons, and this relation is not accidental.

In practice, three types of heat conduction are distinguished: internal heat conduction, external heat conduction, and heat transfer.

2.8.1.1. Internal heat conduction

If there is a constant temperature difference $\Delta T = T_1 - T_2$ between the ends of the metal rod of cross-section A and of length l (Fig. 2.17), then during time Δt, $I = \dfrac{\Delta Q}{\Delta t}$ heat current related to the amount of heat ΔQ passing through any cross section of the rod is proportional to the cross section A and the length l of the rod, and the temperature difference $T_1 - T_2$:

$$I = \frac{\Delta Q}{\Delta t} = -\lambda A \frac{T_1 - T_2}{l},$$

where the negative sign indicates that the heat is always moving to a lower temperature location and λ is the internal thermal conductivity (internal thermal conductivity coefficient) depending on the quality of the material conducting the heat, its unit of measurement is $\dfrac{J}{m \cdot s \cdot K} = \dfrac{W}{m \cdot K}$.

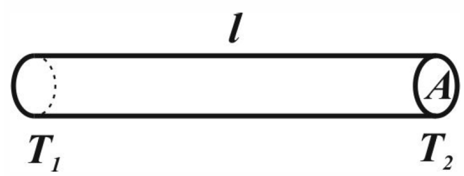

Figure 2.17

The above law on internal heat conduction applies only to stationary (time constant) internal heat conduction. More generally:

$$\frac{dQ}{dt} = -\lambda A \frac{dT}{dx},$$

which also applies to non-stationary cases.

2.8.1.2. External heat conduction

Figure 2.18

If two materials (eg brick and air) are in direct contact with one another on the surface (Figure 2.18) and the contact surfaces are at temperatures T_1 and T_2, then the heat current through the interface is:

$$I = \frac{\Delta Q}{\Delta t} = -\alpha A (T_1 - T_2),$$

where α is the unit of heat transfer coefficient for the two materials, its SI unit is: $\frac{J}{m^2 \cdot s \cdot K} = \frac{W}{m^2 \cdot K}$.

2.8.1.3. The heat transfer

Figure 2.19

The heat transfer through the multilayer medium (Figure 2.19) related to the internal and external heat conduction with a heat current of:

$$I = \frac{\Delta Q}{\Delta t} = -kA(T_1 - T_2)$$

where k is the heat transfer coefficient of the multilayer medium, in SI: $\frac{J}{m^2 \cdot s \cdot K} = \frac{W}{m^2 \cdot K}$

Its value is determined by the thickness of each layer, its internal thermal conductivity and its heat transfer coefficient. For example, double-insulated plastic windows and doors $k \approx 1.4 \frac{W}{m^2 \cdot K}$.

2.8.2. The heat flow

During the heat flow, the actual progressive movement of the material particles conveys heat from the higher temperature to the lower temperature. The heat current $I = \frac{\Delta Q}{\Delta t}$ is proportional to the cross section A perpendicular to the flow direction and to the temperature difference ΔT:

$$I = \frac{\Delta Q}{\Delta t} = -\alpha A \Delta T \ ,$$

where α is the unit of heat flow coefficient in SI: $\frac{W}{m^2 \cdot K}$.

2.8.3. The heat radiation

Heat radiation is a way of spreading heat from one body to another without heating the intermediate medium, without even requiring a transfer medium (see Sun-Earth). Experience has shown that every body irradiates irrespective of the ambient temperature. For example: a person at 36 ^oC radiates heat into the air at 25 ^oC. The heat released by burning the food ensures that it does not cool down. At temperatures below 600 K, the bodies emit infrared (heat) rays. Above this temperature, visible light and even ultraviolet light are emitted.

The spectral energy distribution of radiation, the energy density $\omega(\lambda)$ for a given wavelength as a function of λ wavelength (see chapter "*Nuclear and Particle Physics*") is shown in Figure 2.20.

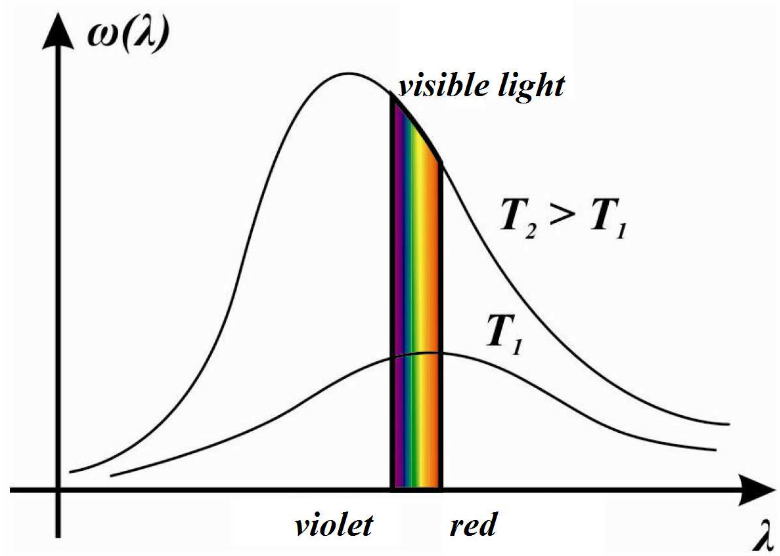

Figure 2.20

The area under the curve gives the numerical value of the total amount of radiated energy. For temperature radiation, the Stefan-Boltzmann Law states that:

"The surface heat flow density of the heat $\frac{1}{A} \cdot \frac{\Delta Q}{\Delta t}$ emitted by the body of surface A is proportional to the fourth power of the surface temperature."

That is, mathematically:

$$\boxed{\frac{1}{A} \cdot \frac{\Delta Q}{\Delta t} = \varepsilon \sigma T^4},$$

where $\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 \cdot K^4}$ and $0 \leq \varepsilon \leq 1$ is the emission factor characteristic of the radiating body, its surface quality, the value of which depends on the smoothness and darkness of the surface.

3. Electromagnetics

Our word for electricity comes from the old Greek equivalent of the word amber (electron). Amber is a fossilized type of resin that is charged by rubbing and goes into electrical state. Today, the electron denotes the negative charges that surround the nucleus. Frictional Electricity has been known for over 2500 years. About the same age is the first experience with magnetism that magnetite ("magnetic iron") attracts tiny pieces of iron. The place where the magnetite was found was the city of Magnesia, hence the name.

No connection or interaction between the two phenomena could be found over the two millennia, the electric and the magnetic states were known to exist separately, independently. The turning point of Alessandro VOLTA, who succeeded in creating a galvanic cell in 1800, which triggered a steady charge current, the electric current, is a very important turning point. We can say that electricity and magnetism are two separate phenomena as long as the charge behavior is constant (static) over time.

The interdependence of the electric field (where electrical phenomena can be observed) and the magnetic field (where magnetic phenomena can be observed) does not occur as long as there is no change in charges or currents over time. The two fields are interdependent if the changes are fast enough. But with that in mind, it's understandable that we're talking about electromagnetism.

Over the past 200 years, important discoveries have been made:

- Coulomb Law on the force between the charges (1785),
- Hans Christian OERSTED's discovery that electricity has a magnetic effect (1820),
- Discovery of André-Marie AMPÉRE on the force between current conductors and the relationship between current and magnetic field (1820, 1825),
- Michael FARADAY's recognition of the real existence of electric and magnetic fields, and his law of induction of a time-varying magnetic field to produce an electric field (1831),
- The Maxwell Equations (1864).

Theoretical and experimental results provided the basis for the conclusion of the 19th century. One of the geniuses of the twentieth century, James Clerk MAXWELL made generalizations that resulted in a system of Maxwell equations that include total electromagnetism. The history of science sees this performance as comparable to Newton's work. The mathematical solution of the Maxwell equations showed that there must be electromagnetic waves propagating in vacuum at the speed of light. The existence of these "predicted" waves was confirmed by an excellent experimental physicist, Heinrich HERTZ, 23 years later. A few decades later, wireless broadcasting, radio and television appeared.

Fate meant that Maxwell did not experience the detection of electromagnetic waves, and that Hertz did not experience the use of electromagnetic waves in communications. Classical electromagnetism has stood the test of time and will always be counted among the highest spiritual achievements of mankind.

Whole electromagnetism can be deduced from the Maxwell equations, but it requires a great deal of sophisticated and deep mathematical knowledge, not to mention physical reasoning. Theoretical electromagnetism follows this path. However, we draw from experience and draw lessons from it. Note that light is also an electromagnetic wave and that Maxwell's equations can be applied to photometric phenomena.

The creation of Classical (Newtonian) mechanics nearly 200 years preceded classical electromagnetism. One of the main reasons for this is that the two types of phenomena are fundamentally different. Mechanical phenomena can be based on everyday sensory experience, whereas we have no sensory experience on electromagnetism (except for static electricity and magnetism), the latter being more abstract and more complicated to perform experiments and measurements.

3.1. Electrostatics

3.1.1. Basic concepts of electrostatics

Electrostatics deals with the phenomena of dormant electrical charges. Certain building blocks of atoms are electrically charged particles. These are the electron (negative charge) and the proton (positive charge). These are also considered as elementary charges because fractions of their charge do not occur in individual particles. Because bodies generally have equal numbers of electrons and protons, bodies are electrically neutral. If a body is not electrically neutral then

- negatively charged when electron excess is present, and
- positively charged when electron deficit is present.

The law of charge retention states that in a closed system the amount of electrical charge remains unchanged.

The degree of charge is denoted by Q (sometimes q) and its SI unit is coulomb, denoted by C. The electric charge is a signed quantity. A 1 C charge corresponds to $6.28 \cdot 10^{18}$ elemental electrical charges, which means that the value of the elementary charge is:

$$\boxed{e = \frac{1}{6.28 \cdot 10^{18}} C \approx 1.6 \cdot 10^{-19} C},$$

where e is the distinguished sign of the elemental electric charge.

On a macroscopic scale, it is expedient to consider a set of charges as having a continuous distribution and talking about charge density.

3.1.1.1. Linear charge density

On a one-dimensional body, we talk about linear charge density λ, which in the case of a uniform charge distribution:

$$\lambda = \frac{Q}{l},$$

where l is the length of the linear body with Q charge. SI unit: $\frac{C}{m}$. In the case of inhomogeneous charge distribution, the local charge density λ at a given location:

$$d\lambda = \frac{dQ}{dl}.$$

3.1.1.2. Surface charge density

Two-dimensional, on surface A, we refer to the surface charge density σ, whose magnitude at uniform charge distribution is:

$$\sigma = \frac{Q}{A},$$

whose SI unit is $\frac{C}{m^2}$. In the inhomogeneous case, the local surface charge density σ is:

$$d\sigma = \frac{dQ}{dA}.$$

3.1.1.3. Volumetric charge density

On a three-dimensional V volume body, we talk about the volumetric charge density ρ:

$$\rho = \frac{Q}{V}$$

in a homogeneous case, and

$$d\rho = \frac{dQ}{dV}$$

for inhomogeneous charge distribution. Its SI unit is $\frac{C}{m^3}$.

3.1.1.4. Point charge and conductor

It will also be advisable to introduce a point charge when the size of the charge carrier is negligible compared to its surroundings.

The materials in which electrical charges can move relatively freely are called conductors. For example, metals where the charge carriers are electrons and electrolytes where the charge carriers are ions. Insulators are materials in which the charge carriers can move only very limited, the charge applied to the surface of such materials remains in place and is not distributed.

3.1.2. The Coulomb Law

Charles Augustin de COULOMB studied the force acting on electric charges (1785). He found that:

"The electric force acting between the point charges Q_1 and Q_2 is directly proportional to the magnitude of the charges, inversely proportional to the square of the distance r between them, and the force falls in the line connecting the charges."

That is, mathematically:

$$\boxed{F_C = k\frac{Q_1 Q_2}{r^2}},$$

where the proportionality factor k is a constant depending on the choice of base units, in our case $k = 9 \cdot 10^9 \frac{N \cdot m^2}{C^2}$. This is the Coulomb Law. The mathematical form of the law also reflects the experiential fact that charges of the same sign repel and of different sing attract each other.

It is legitimate to assume that the point charge exerts an electric attractive or repulsive force in a spherical symmetry regardless of direction; therefore, for convenience, we use the $k = \frac{1}{4\pi\varepsilon_0}$ notation, where 4π is the size of the total space angle (numerically the surface of a sphere of unit radius), of which:

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \cdot 10^{-12} \frac{C^2}{N \cdot m^2},$$

the permittivity of the vacuum (≈ air) (formerly called dielectric constant). The ε permittivity of insulating materials other than vacuum is usually given by the relative permittivity ε_r relative to the vacuum, of which the

$$\varepsilon = \varepsilon_r \varepsilon_0$$

permittivity of the given material can be calculated. For example, for glass, $\varepsilon_r = 5$, so the permittivity of the glass is:

$$\varepsilon = \varepsilon_r \varepsilon_0 = 5 \cdot 8.85 \cdot 10^{-12} \frac{C^2}{N \cdot m^2} = 4.42510^{-11} \frac{C^2}{N \cdot m^2}.$$

Experience has shown that although $\frac{1}{r^2}$ increases rapidly with the decrease of r, the Coulomb Law remain highly accurate even at atomic distances (10^{-10} m). The electric force at atomic dimensions is many orders of magnitude greater than the gravitational force when compared at the same distance between the same two bodies. Therefore, where there are electric forces, gravitational attraction is generally negligible.

3.1.3. The electric field and its characterization

The part of the space that exerts force on the resting electric charge placed there is called an electric field. To our knowledge, such a field is created either by electrical charges (in this case, the sources of the field) or, in time, by a variable magnetic field (in which there are no sources, see below).

In electrostatics, we only deal with the electric field generated by resting charges. The electric field is one of the forms of appearance of matter, which carries energy and momentum, and is able to exert a force on the Q charge placed there. The electric field – its points – is numerically characterized by the force exerted on a positive charge of the unit placed there. So the electric field is a vector space, for each of its points (location vectors) a vector called electric field strength vector $\vec{E}$ can be assigned to. As mentioned above:

$$\boxed{\vec{E} = \frac{\vec{F}_e}{Q}},$$

where Q is the charge placed at a given point in space and $\vec{F}_e$ is the force acting on it. The unit of electric field strength in SI is $\frac{N}{C}$ $(= \frac{V}{m}$, see later).

In electrostatics, $\vec{E}$ depends only on location, not time. If $\vec{E}$ is not a function of the location, that is, the magnitude and direction of the electric field are the same at all points in the space, then it is homogeneous, otherwise it is an inhomogeneous electric field.

The electric field can be visualized by means of field strength lines or Faraday force lines such that:

- The field strength vector should be in the direction of a tangent drawn to the field strength line at a given point and should be in the same direction as the $\vec{F}_e$ electric force acting on the positive charge.
- The number of force lines passing through a unit surface perpendicular to the field strengths is equal to the magnitude of the field strength.
- The electric field of a positive point charge can be illustrated by force lines radiating outwards from the charge, and the electric field of a negative point charge by force lines radiating symmetrically towards the charge (Figure 3.1):

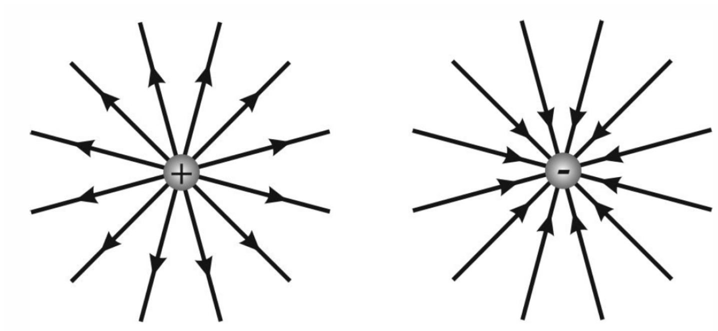

Figure 3.1

3.1.4. The behavior of the electric dipole in a homogeneous electric field

Connected $+Q$ and $-Q$ charges that are spaced apart at l distance are called electrical dipoles. An electric dipole placed in a homogeneous electric field in a stationary position is only affected by a torque M, because $\vec{F}_+ + \vec{F}_- = 0$, the resultant of the forces acting on it is zero, so it does not make any translational motion (Figure 3.2).

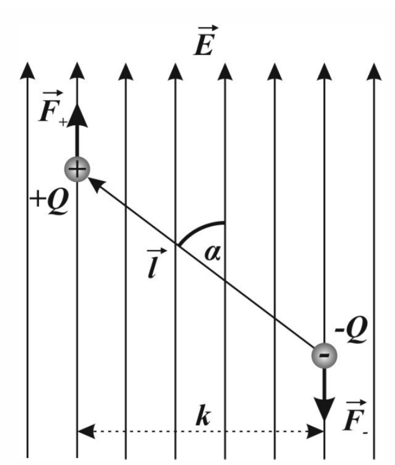

Figure 3.2

The torque of the force pair $F_+ = F_- = QE$ acting on the charges is

$$M = Fk = Fl \sin\alpha = QEl \sin\alpha .$$

The product of Ql is called the p electric dipole moment of the dipole:

$$\boxed{\vec{p} = Q\vec{l}},$$

so the torque is:

$$M = pE \sin\alpha .$$

The dipole moment is a vectorial quantity, its direction lies in the line connecting the two charges and points from the negative charge to the positive charge. With all this in mind, the torque vector is a vectorial product:

$$\boxed{\vec{M} = \vec{p} \times \vec{E}}$$

The torque M rotates the dipole in the electric field. If the electric field is inhomogeneous, the dipole will rotate and move towards higher field strengths.

Comment:

Many molecules have an electric dipole moment. If these so-called. polar molecules can move freely (eg in gases), they can be separated from the rest of the gas by using an inhomogeneous electric field (eg electrostatic precipitators).

3.1.5. The flux of the electric field

The electric flux Ψ is defined as the number of force lines perpendicular to the surface A perpendicular to the electric field. Since the electric field E gives a numerical value of the number of force lines passing through a unit surface, in a homogeneous electric field EA force lines are passing through a perpendicular surface A, so in this case the flux of the electric field:

$$\boxed{\Psi = EA}.$$

The SI unit of electric flux is: $\dfrac{N \cdot m^2}{C}$ $(= V \cdot m$, see later).

In the case of a surface of any shape, the surface is subdivided into surface elements dA so small that the electric field can be considered homogeneous there. A surface element of size $d\vec{A}$ is considered to be a vector quantity such that its length is proportional to the size of dA and its direction is equal to the direction of a surface normal vector of unit length $\vec{n}$ perpendicular to the surface of the element (Figure 3.3):

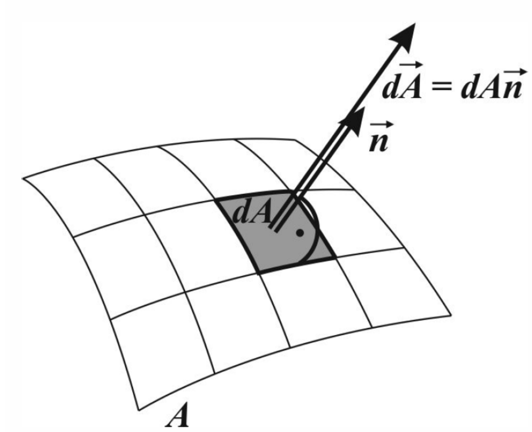

Figure 3.3

Then the elemental flux $d\Psi$ of the electric field E passing through the surface of dA and its angle α with its surface vector is calculated as the product of the dA surface element and the component of the field strength vector E facing the surface $\vec{n}$:

$$d\Psi = E_n dA = EdA\cos\alpha = \vec{E} \cdot d\vec{A},$$

that is, given by the scalar product of the surface element vector $d\vec{A}$ and the electric field strength vector $\vec{E}$ (Figure 3.4):

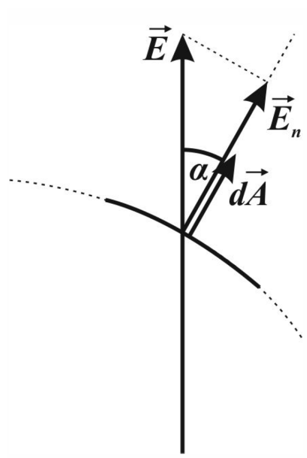

Figure 3.4

The total flux calculated for surface A is obtained by integrating the elemental fluxes:

$$\boxed{\Psi = \int_A \vec{E} \cdot d\vec{A}}.$$

Calculate the flux of the electric field of the point charge Q on a spherical surface of radius r concentric to it. As we have seen, on a spherical surface of radius r measured from charge Q, the direction of electric field strength is perpendicular to the surface of the sphere and its magnitude is

$$E = E_n = k\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}.$$

Of which the electric field flux:

$$\Psi = \int_A \vec{E}\cdot d\vec{A} = \int_A E_n dA = E_n \int_A dA = k\frac{Q}{r^2}4\pi r^2 = 4\pi kQ,$$

by using $k = \frac{1}{4\pi\varepsilon_0} \to 4\pi k = \frac{1}{\varepsilon_0}$

$$\Psi = \frac{Q}{\varepsilon_0}.$$

The example above is generalized by Gauss's law:

"The flux Ψ of the electric field of all charges $\sum Q$ in a space enclosed by any closed surface is always $\Psi = \frac{\sum Q}{\varepsilon_0}$ on the closed surface, regardless of the distribution of all charges $\sum Q$ within the space."

That is, mathematically:

$$\boxed{\Psi = \oint_A \vec{E}\cdot d\vec{A} = \frac{\sum Q}{\varepsilon_0}}.$$

The Gauss's law is Maxwell's 1st equation of the unified electromagnetic theory, which describes the flux of a static electric field in vacuum, or the so-called source of strength. The law, in words:

"The electrostatic field has source, the sources are the electric charges, of which the electric force lines start from and go to infinity."

3.1.6. Work and voltage of the electric field

3.1.6.1. Work

We have seen that the electric field exerts a force $\vec{F} = Q\vec{E}$ on the charge, which causes the charge to move, ie the electric field can work on it. The magnitude of the work performed in a time-constant, inhomogeneous field is shown in Figure 3.5. using chapter "Mechanics".

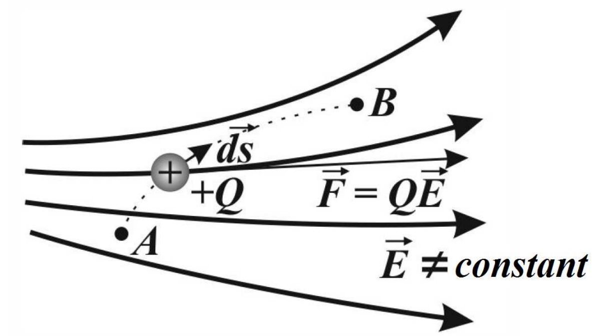

Figure 3.5

Due to the inhomogeneity of the field, the electric force $\vec{F} = Q\vec{E}$ acting on the point charge Q moving on the AB path is variable in magnitude and direction, so the charge path is divided into elementary $d\vec{s}$ displacements where the electric field can be considered homogeneous with small error. Then the electric field of strength $\vec{E}$ on charge Q at displacement $d\vec{s}$ performs

$$dW = \vec{F} \cdot d\vec{s} = Q\vec{E} \cdot d\vec{s}$$

elementary work. All electrical work on the charge moving along the path between points A and B is the sum of the elementary work, that is, the line integrate of the electric field strength vector $\vec{E}$ along the path:

$$\boxed{W_{AB} = Q\int_A^B \vec{E} \cdot d\vec{s}}.$$

In homogeneous field and on straight path

$$W_{AB} = Q\vec{E} \cdot \vec{s} = QEs\cos\alpha ,$$

and if $\vec{E}$ and $\vec{s}$ are unidirectional ($\alpha = 0^o$) then

$$W_{AB} = QEs .$$

3.1.6.2. Voltage

The work done by the electric field is not only a characteristic of the electric field, as its extent also depends on the magnitude of the charged Q. Therefore, we introduce the

$$U_{AB} = \frac{W_{AB}}{Q}$$

quantity, which in turn is only a quantity specific to the field, because it shows numerically the amount of work done per unit charge between points A and B, called voltage. Say:

$$\boxed{U_{AB} = \frac{W_{AB}}{Q} = \int_A^B \vec{E} \cdot d\vec{s}},$$

Its SI unit is $\frac{J}{C} = V$ (volt). If the electric field is homogeneous and the path is straight, then:

$$U_{AB} = \frac{W_{AB}}{Q} = \vec{E} \cdot \vec{s} = Es\cos\alpha ,$$

and if $\vec{E}$ and $\vec{s}$ are unidirectional ($\alpha = 0°$) then

$$U_{AB} = Es .$$

This results in a more frequent use of the unit of electric field strength: because of $E = \frac{U_{AB}}{s}$,

the SI unit can be written in the form of $\frac{N}{C} = \frac{V}{m}$.

3.1.6.3. Accelerating effect of electric field

Since the homogeneous electric field of voltage U performs $W = QU$ work on the particle of initial velocity v_0 of mass m, and of charge Q, that is, it accelerates to velocity v, according to the work theorem, and increases the kinetic energy:

$$W = QU = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 .$$

Comment:

It is not advisable to give the energy of elementary particles in joules because it is too large there. Therefore, we introduce the electron volt (eV) energy unit acquired by a stationary particle with elementary charge ($e = 1.6 \cdot 10^{-19} C$*) when passing through an electric field with a voltage of 1 V. The magnitude of this in SI:*

$$1eV = QU = eU = 1.6 \cdot 10^{-19} C \cdot 1V = 1.6 \cdot 10^{-19} J .$$

3.1.7. Potential energy and potential

Like the potential energy described in Mechanics, we introduce the potential energy of the Q charge in the electric field.

As a reminder, the potential energy of a body slowly moved by a against gravity is equal to the work done by the force of lifting the body to a height above a designated reference point on the Earth's surface.

In the electric field, the potential energy of the charge Q equals to the work performed by a driving force that has opposite direction to the electric force, when moving the point charge slowly from the designated reference point $A = 0$ to a given point:

$$E_p = W_{driving} = -W_{electric},$$

that is,

$$\boxed{E_{p,B} = -W_{A=0,B} = -Q \int_{A=0}^{B} \vec{E} \cdot d\vec{s}}.$$

Comment:

Do not confuse the markings on the left and right of the context. E on the left indicates energy and on the right the electric field strength!

Similarly to work, the potential energy value is not limited to the electric field, since its magnitude depends on the amount of the given point charge. Therefore we introduce

$$\boxed{V_{0,B} \equiv V_B = \frac{E_{p,B}}{Q}},$$

the potential of the point B of the electric field, which is numerically equal to the potential energy of the unit charge at point B.

It can be proved that the electrostatic field is conservative, which means that the work done to move the Q charge from point A to point B is independent of the shape of the path, but depends on the location of the start and end points. It follows that the work done along a closed curve is zero because the magnitude of the work from A to B is the same as from B to A but has the opposite sign (Figure 3.6).

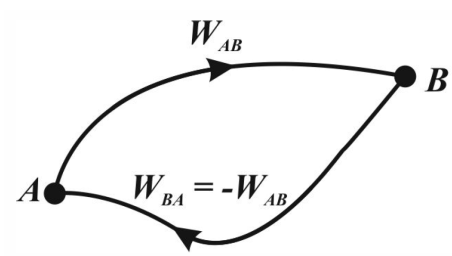

Figure 3.6

Say:

$$W_{AA} = W_{AB} + W_{BA} = W_{AB} + (-W_{AB}) = 0.$$

By moving the Q point charge in an electrostatic field along a closed curve, the work in general:

$$W = Q\oint \vec{E} \cdot d\vec{s} = 0,$$

therefore the so-called circular voltage for a closed curve:

$$\boxed{\oint \vec{E} \cdot d\vec{s} = 0}.$$

This is the space equation of Electrostatics because it only contains the properties of the electric field. An important feature of the electrostatic field here is that there are no closed force lines, in other words, it is a vortex-free vector space.

The points with the same potential are located on a surface, called an equipotential surface, by which the electric field can be illustrated as well as the force lines. The tangents of the force lines are perpendicular to the tangent plane of the equipotential surface.

Knowing the concept of potential, the U_{AB} voltage between point A and point B of the electric field can be interpreted differently. The voltage between two points in the field equals the amount of work required to transfer a positive unit charge from one point to another, in other words, the potential difference between the two points.

The numerical value of the potential is the numerical value assigned to a point in the field, while the voltage is always interpreted between two points.

3.1.8. Electrical capacity, capacitors

Two conductors separated by an insulator form a capacitor. The two conductors (usually surfaces) are the plates ("armaments") of the capacitor. Applying a charge of $+Q$ to one of the conductors and to $-Q$ of the other leads to the formation of an electric field with a voltage of U (Figure 3.7):

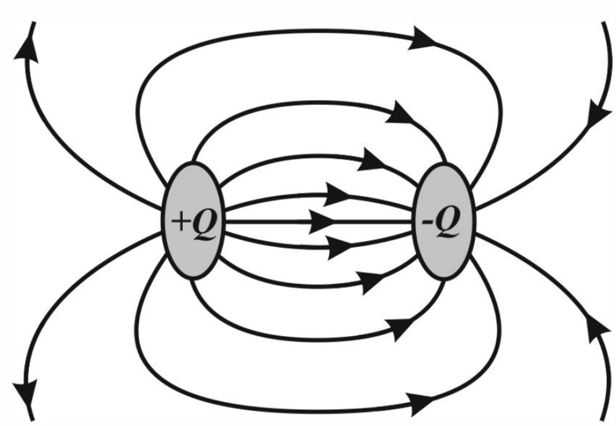

Figure 3.7

Experience has shown that for a given spatial arrangement, the quotient $\frac{Q}{U}$ is constant and is called the system capacity C:

$$\boxed{C = \frac{Q}{U}}.$$

Its SI unit is $\frac{C}{V} = F$ (farad). In practice, fractions of the farad are used: mF, μF, nF, pF. The capacitor is capable of storing charge at a given voltage.

Determine the capacity of the capacitor located at a distance d, with $+Q$ and $-Q$ charged A surfaces, if there is air between them.

The electric field between the armaments is homogeneous and has a field strength of

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{\sum Q}{\varepsilon_0}$$

Determined by Gauss's law. From the equation because of symmetry and homogeneity

$$EA = \frac{Q}{\varepsilon_0},$$

that is,

$$E = \frac{Q}{\varepsilon_0 A}$$

so the amount of the charge stored on the armaments is:

$$Q = E\varepsilon_0 A.$$

By definition of voltage:

$$U = Ed.$$

Using the previous two results is the capacity of the capacitor is:

$$C = \frac{Q}{U} = \varepsilon_0 \frac{A}{d}.$$

Capacity can be increased by increasing the size of the plates, reducing the distance between them, and using insulators with a higher permeability than air (vacuum) (Figure 3.8).

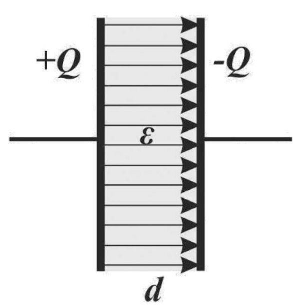

Figure 3.8

Technical drawing of the capacitor:

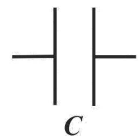

Figure 3.9

3.1.8.1. Connecting capacitors

Capacitors can be connected to a simpler or more complex network. When connected, all of them can be replaced by a single capacitor with a resulting capacity of C_r.

3.1.8.1.1. The resulting capacitance of the capacitors connected in parallel

Parallel connection occurs when the armaments of each capacitor are connected to one another by an intermediate branch node, not a continuous wire (Figure 3.10).

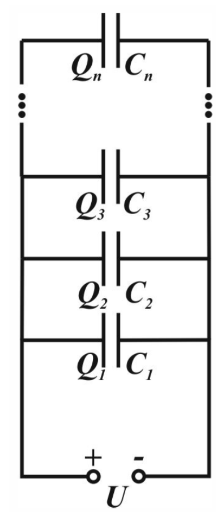

Figure 3.10

In the case of parallel connection, the poles of the voltage source and the poles of the armaments of each capacitor on the same side are equipotential points, so the voltage between the capacitors' armaments is the same as the voltage of the voltage source.

Therefore, the resulting capacity:

$$\boxed{C_r = \frac{\sum Q}{U} = \frac{Q_1 + Q_2 + \ldots + Q_n}{U} = C_1 + C_2 + \ldots + C_n},$$

the sum of the capacities of each capacitor.

3.1.8.1.2. The resulting capacity of the capacitors connected in serial

Serial connecting occurs when the armaments of each capacitor are connected to each other by a continuous wire without an intermediate branch node (Figure 3.11).

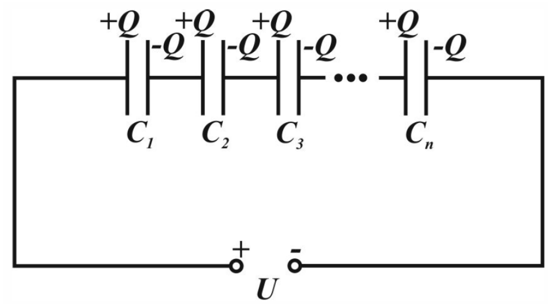

Figure 3.11

In the case of serial connection, the capacity of each capacitor will be the same (due to the electric division) the charge ($+Q$ and $-Q$ per armament). The sum of the voltages applied to them is equal to the voltage U provided by the voltage source according to the law of energy conservation:

$$U = U_1 + U_2 + \ldots + U_n,$$

from where using $U = \dfrac{Q}{C}$ form $C = \dfrac{Q}{U}$:

$$\frac{Q}{C_r} = \frac{Q}{C_1} + \frac{Q}{C_2} + \ldots + \frac{Q}{C_n},$$

and we get a relationship for the resulting capacity:

$$\boxed{\frac{1}{C_r} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}}.$$

3.1.9. Piezoelectric effect

Some crystals (quartz, tourmaline, etc.), by virtue of their crystal structure, have properties that, when properly deformed, produce a polarization surface charge density at their interfaces and change sign in the direction of deformation force. This phenomenon is called direct piezoelectric effect. By placing the crystal between the plates of the capacitor, the voltage on the capacitor can be measured. For example, the crystal microphone and pressure gauges work on this principle.

The reverse is the reciprocal piezoelectric effect. A piezoelectric crystal placed in an electric field (between capacitor elements) undergoes a deformation when the voltage is applied. A

Fourier transform of a rectangular voltage pulse of single width *Δ* contains all frequencies, including the resonance frequency of the crystal, in a given bandwidth. Such a voltage pulse thus causes the piezoelectric crystal to vibrate, the frequency of which depends on the thickness of the crystal and is therefore capable of emitting ultrasound, for example.

3.2. Charge flow, electric currents

3.2.1. Electric current

The unidirectional movement of charges is called electric current. In practice, the ordered movement of a plurality of charges (electrons, ions) is called electric current. In metallic wires, current is created only by the movement of electrons, because metals contain easily displaceable (stationary) electrons.

The charge movement, i.e. the electric current, is related to the voltage characteristic of the electric field between the two points of the conductor. The electric field exerts a driving force on the conductor charges. The device that generates an electric field and the characteristic voltage between the two points of the conductor is called a power source.

Comment:

The name of the voltage source is also considered appropriate. In electrical engineering, however, the voltage generator is distinguished from the current generator.

The corner (pole) of a power source where there is an excess of electrons is called negative, where there is a lack of electrons, positive. Since the electric field between the two poles starts from the positive charge and ends at the negative charge, the field strength of the electric field of the power source points from the positive pole to the negative pole. In a closed circuit, electrons with a negative charge move from the negative pole to the positive pole, this is called the physical current direction (Figure 3.12).

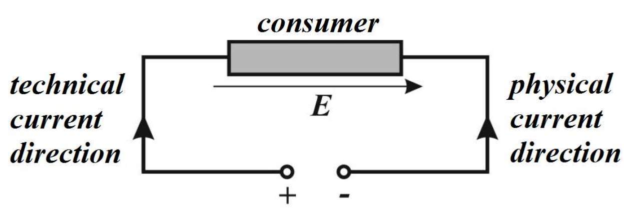

Figure 3.12

For historical and practical reasons, the direction of movement of positive charges is called the technical current direction. The two types of currents are opposite, they are equal in magnitude, and the two are equivalent. We will now use the technical current direction.

Power sources convert some kind of energy into electrical energy: mechanical energy generators, chemical energy galvanic cells, batteries, light energy photovoltaic cells, heat thermocouples, etc. Constant electrical current can only flow in a closed circuit consisting of a power source, a consumer (which converts electrical energy into other energy, such as heat, light, mechanical, etc.) and connecting wires (which, ideally, do not consume electrical energy). Common technical designation of power sources:

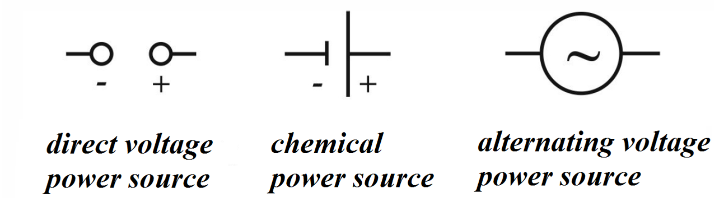

Figure 3.13

Inside the conductor, the electric field exerts an electric force $\vec{F}_e = Q\vec{E}$ on the charges. In a metallic conductor, positively charged charges with elastic forces cannot move in the metal lattice (apart from their vibratory motion), but electrons can move relatively freely. The electrons are accelerated by the electric field in the opposite direction of the field strength until they are collided with the stationary positive ions. As a result, their speed is slightly reduced, the kinetic energy is reduced to heat and then accelerated again until the next collision. By examining many electrons in the wire, it can be shown that they obtain an average velocity $\langle v \rangle$ of approximately $1.8\frac{m}{h}$ in metals. The ionic lattice structure thus somewhat impedes the movement of electrons and exhibits resistance to motion. The collision of electrons with each other is negligible compared to the collision with ions. The "lost" energy is replaced by the power source, ie it supplies power to the circuit (for example, galvanic cells become exhausted over time and their charge separation effect is greatly reduced). Electric current, as a charge movement, is characterized by amperage and current density.

3.2.1.1. Amperage

The amperage I is the quotient of the charge Q through the whole cross-section of the conductor and the time t required. If the charge flow is constant over time, then

$$\boxed{I = \frac{Q}{t}},$$

and talking about direct current (DC), its SI unit is $\frac{C}{s} = A$ (amperes). Its numeric value shows the charge flowing over the entire cross-section of the conductor over a unit time. The variable current is denoted by the letter i. If the charge current is not uniform, then the average current for a short period of time Δt is:

$$\langle i \rangle = \frac{\Delta Q}{\Delta t}.$$

The limit of this is the instantaneous current:

$$\boxed{i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = \dot{Q}(t)}.$$

3.2.1.2. Current density

If the charge flow distribution in the conductor is equal to the direction of travel in the whole cross section A, then at the current density J

$$\boxed{J = \frac{I}{A}}$$

is a quotient whose numerical value indicates the charge amount flowing through the unit cross-section of the conductor over unit time. SI unit: $\frac{A}{m^2}$.

If the charge flow is not uniformly distributed along the cross-section, then the current density at a given point will be similar to the previous pattern:

$$j = \frac{dI}{dA}.$$

3.2.2. The Ohm's law for a homogeneous conductor and electrical resistance

3.2.2.1. Electrical resistance

According to Georg Simon OHM studies, the U voltage per consumer and the current I through it are directly proportional, ie their ratio is constant, denoted by R and is called the electrical resistance of the consumer:

$$\boxed{R = \frac{U}{I}}.$$

The SI unit of resistance is: $\frac{V}{A} = \Omega$ (ohm).

Quantity G is called electrical conductivity and its SI unit is $\frac{1}{\Omega} = S$ (siemens). The resistance and the electrical conductivity are not properties of the quantities of electricity that cause the charge flow, but of the material (wire, consumer) providing the movement of the charges. We have already indicated that the movement of charges in the circuit is hindered to a greater or lesser extent by wires, but mainly by consumers. An ideal conductor is one that does not have such a hindrance, such as superconductors.

Experience has shown that the resistance R of a homogeneous material (wire, wire) is proportional to the length l of the wire and inversely proportional to its cross section A. calculation:

$$R = \rho \frac{l}{A},$$

where the proportionality coefficient of the substance ρ is the so-called. specific resistance, unit in SI: Ωm (in practice $\Omega \frac{mm^2}{m}$ is used). We also use the word resistance to describe two things:

- The property of the conductive material to resist the flow of electric charges.
- A resistor R in the circuit is also referred to as a resistance.

The inverse of the specific resistance is the specific conductivity (conductance):

$$\gamma = \frac{1}{\rho},$$

whose SI unit is: $\frac{1}{\Omega m} = \frac{S}{m}$.

Comments:

1. From the above it can be concluded that the resistance is temperature dependent (think of the thermal motion of the ions). Experience has shown that at a given temperature T the electrical resistance:

$$R_T = R_0[1 - \alpha(T - T_0)],$$

where R_0 is the resistance measured at temperature T_0, α is the material constant, the temperature coefficient of resistance, the unit of measure of which in SI is $\frac{1}{K} = \frac{1}{°C}$. By using the temperature dependence of the resistance it is possible to make a thermometer.

2. In chemistry, the molar conductivity Λ_m *of electrolytes is defined as the quotient* $\Lambda_m = \frac{\lambda}{c}$*,*

where c is the concentration of matter: $c = \frac{n}{V}$*. SI unit:* $\frac{Sm^2}{mol}$.

3.2.3. Complex circuits, Kirchhoff's laws

3.2.3.1. Kirchhoff's 1st Law

A complex circuit (network) can have more consumers (resistors) and multiple sources of power. The intersection of more than two wires is called a node. The consequence of the charge retention law is Kirchhoff's 1st law, or so-called node law that states:

"The sum of the currents entering the node is equal to the sum of the currents exiting the node."

That is, mathematically:

$$\sum I_{in} = \sum I_{out}$$

If the current currents are marked with a + sign and the currents are marked with a – sign, Kirchhoff's 1st Law can also be written as:

$$\boxed{\sum I = 0}.$$

3.2.3.2. Kirchhoff's 2nd Law

The circuit section between nodes is called a branch. Closed circles made up of branches are loops. Kirchhoff's 2nd or so-called loop law expresses the law of energy conservation and states that:

"In a closed circuit (loop), the sum of the voltages U of the power sources is equal to the sum of the voltages U_R per resistor (consumer).

So mathematically:

$$\sum U = \sum U_R .$$

The sign of the voltage of the power sources is related to the direction of their electric field. Since the electric field strength is from the positive pole to the negative pole, so is the voltage from the positive pole to the negative pole, but remember that it is a scalar physical quantity. Voltage to the consumer is considered positive if the current flows in the same direction as the positive bypass, which is arbitrarily assigned by us, otherwise it is negative. With this in mind, Kirchhoff's 2nd Law can be written in the form below as well:

$$\boxed{\sum U = 0}.$$

3.2.4. Connecting consumers

In the case of connecting of consumers, consumers can be replaced by a single resistor R_r, by which an electric current of the same magnitude can be observed in the circuit. Consumers, like capacitors, can be connected in serial and in parallel.

3.2.4.1. Serial connection

Connecting consumers without a branch junction is called serial connection (Figure 3.14). The resistance of series connected consumers can be replaced by a single resistor, which uses a current of the same amperage from the power source.

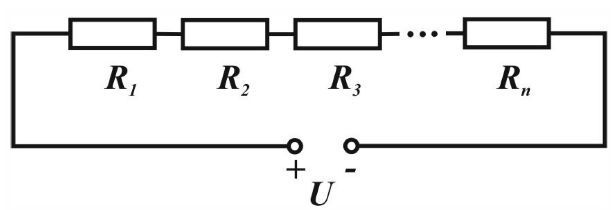

Figure 3.14

The voltage across the R_r resistor using Ohm's law:

$$U = IR_r$$

According to Kirchhoff's 1st law, all consumers connected in series without a branch junction have the same amperage I and according to Kirchhoff's 2nd law, the sum of the voltages for each resistor is equal to the voltage of the power source:

$$U = U_1 + U_2 + \ldots + U_n = IR_1 + IR_2 + \ldots + IR_n .$$

Comparing this to the previous result, in the case of serial connection, the resulting resistance of the consumers is the sum of the individual resistors:

$$\boxed{R_r = R_1 + R_2 + \ldots + R_n}.$$

3.2.4.2. Parallel connection

Parallel connection is used when the terminals of each consumer are connected to each other by an intermediate branch junction (Figure 3.15).

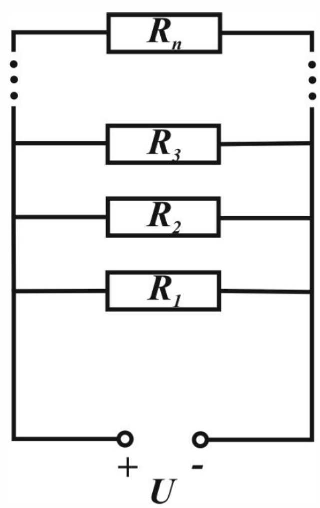

Figure 3.15

The resistance of parallel connected consumers can also be replaced by a single resulting resistance. Then the pole on the given side of the power source and the terminals on the same side of the consumers are equipotential points, so the voltages to the consumers are of the same magnitude and the same as the voltage of the power source.

The current flowing through a resistor with a resistance R_r, replacing the resistance of the connected consumers, using Ohm's law:

$$I = \frac{U}{R_r}.$$

According to Kirchhoff's 1st law, the current flowing in a branch immediately below the source of power (hereinafter "the main branch") is equal to the sum of the currents flowing in the individual branches:

$$I = I_1 + I_2 + \ldots + I_n = \frac{U}{R_1} + \frac{U}{R_2} + \ldots + \frac{U}{R_n}.$$

Comparing this to the previous result, in the case of parallel connection, the reciprocal of the resulting resistance of the consumers is equal to the sum of the reciprocals of the individual resistors:

$$\boxed{\frac{1}{R_r} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}.$$

3.2.4.3. Combined connection

In more complex circuits, we also see a combination of serial and parallel circuits. Such couplings are called mixed connection. Such an application is for example with a loaded voltage divider potentiometer. The potentiometer is made by terminating one of two resistors connected in series (Figure 3.16):

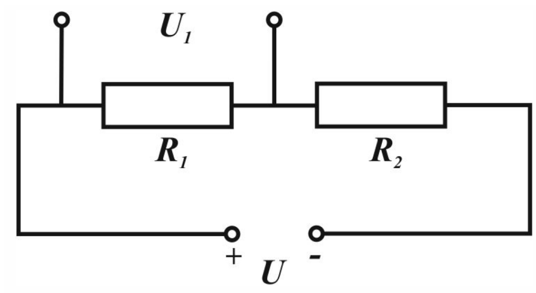

Figure 3.16

In this case, according to the above the voltage of the terminal is less than the voltage of the power source U

$$U_1 = U \frac{R_1}{R_1 + R_2}$$

A more convenient version of the potentiometer is that the slider R_1 can be used to continuously change the settings as shown in Figure 3.17.

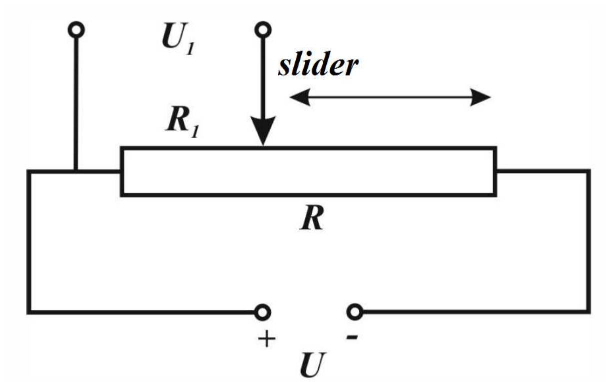

Figure 3.17

Then the voltage measured at the terminals

$$U_1 = U \frac{R_1}{R}$$

can be continuously changed by sliding the slider and can therefore be set between $0 < U_1 < U$ (unloaded potentiometer).

If the split voltage is to be used to supply an R_t resistive load (loaded potentiometer), the situation is more complicated (Figure 3.18).

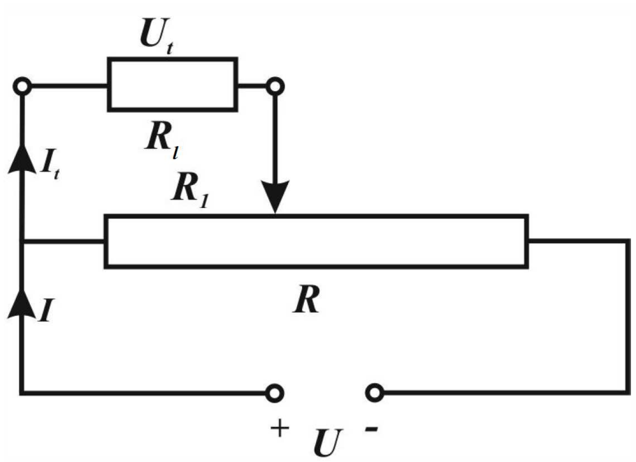

Figure 3.18

Note that R_1 and R_l are connected in parallel, whereas they resulting is in serial with $R - R_1$ that is smaller than R. Now the split voltage is:

$$U_l = U \frac{\dfrac{R_1 R_l}{R_1 + R_l}}{\dfrac{R_1 R_l}{R_1 + R_l} + \left(R - R_1\right)}$$

It can be deduced that in the case of $R_l \to \infty$ we return to the so-called unloaded state. Here again, depending on the position of the slider and at the same setting, the voltage measured U_l under load is less than the value measured under unloaded condition.

Comment:

The potentiometer discussed above is continuously adjustable, so-called linear potentiometer because the distribution of resistance changes proportionally with the slider displacement. There are non-linear potentiometers (eg logarithmic) where moving the slider distribution of resistance is not proportional but by some function. Volume controls are usually logarithmic potentiometers.

3.2.5. Amperage and voltage measuring instruments

The current flowing through the consumer can be measured by an ammeter connected in series with the consumer to measure the current flowing in the same branch as the consumer. To do this, the circuit must be interrupted before the instrument can be connected. The ideal (internal) resistance of the ammeter is zero so that the current in the circuit that contains the ammeter is not changed (Figure 3.19):

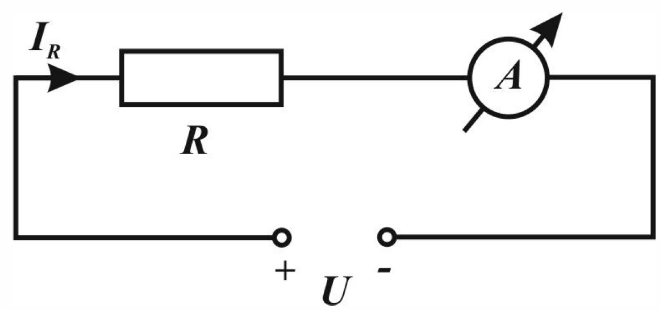

Figure 3.19

Voltage can be measured with a voltmeter connected in parallel with the consumer without the need for circuit opening. The ideal (internal) resistance of an ideal voltmeter is infinitely high so that no current can flow through it (Figure 3.20):

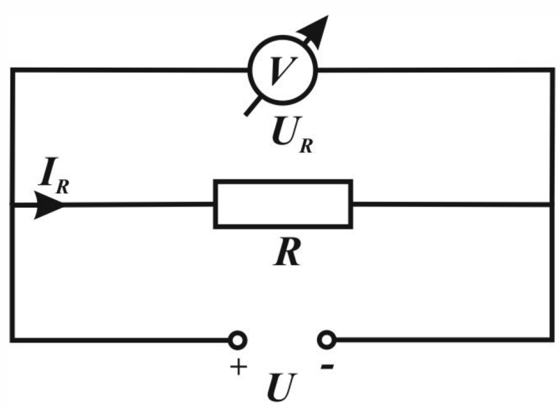

Figure 3.20

3.2.6. The work and performance of direct current

3.2.6.1. The work of direct current, the Joule heat

The electric field of voltage U performs work on charge Q is $W = QU$. Applying the $Q = It$ relation gives

$$\boxed{W = UIt}$$

Using Ohm's law, the work of the I direct current on the resistor R can also be written as

$$W = I^2 Rt = \frac{U^2}{R} t$$

According to the work theorem, the free charge carriers moving in a resistor R acquire kinetic energy due to the electric field, but they also continuously release the kinetic energy obtained during their frequent collisions with the ions in the grid. As a result, the consumer's internal energy increases (heats up) and then releases to the environment. The increase in internal energy lasts until the heat generated by the consumer, the so-called Joule heat, matches the work of the electric current generated by the electric field. Thus, the Joule heat generated in the consumer:

$$\boxed{Q = UIt = I^2 Rt = \frac{U^2}{R} t}.$$

This is manifested by the heat effect of the electric current. It can be seen that at high I current, due to the square proportionality, it can produce very high Joule heat, which can be detrimental to energy transport (eg power lines) but useful for utilizing the heat of electric current (eg radiators).

3.2.6.2. The performance of direct current

Performance is the quotient of work and time, so the performance of a direct current flowing through a consumer of U voltage and R resistance is:

$$\boxed{P = \frac{W}{t} = UI = I^2 R = \frac{U^2}{R}}.$$

3.3. Magnetostatics

3.3.1. Basic concepts of magnetostatics

Magnetostatics investigates the properties of a permanent magnetic field over time. In time, there is a permanent magnetic field for the permanent magnets (rod magnets, horseshoe magnets) and the direct currents discussed in the previous chapter.

The essence of electrical and magnetic phenomena has been unknown for millennia, and many similarities have been discovered between the two. Therefore, the laws of magnetostatics (eg the magnetic "Coulomb's law") have been developed for the analogy of Electrostatics. The magnet rod behaves like an electric dipole in terms of force, so it was expedient to introduce two different magnetic poles, N (North) and S (South), analogous to Earth as magnet. However, there is a fundamental difference between the electric and the magnetic poles: the positive and negative charges of the electric dipole can be separated, but splitting the magnetic rod always produces a full, two-pole magnet – the dipole – that is, there are no true magnetic monopoles.

The iron placed near the magnet also acts as a magnet (attracting the other iron), a phenomenon of magnetic division. This can be explained by the fact that iron has disordered magnetic regions, which are arranged by their external action and thus act as a bipolar magnet (dipole). On the iron side near the magnet, there is an opposite pole to the magnet, so we always have an attractive effect. If the magnet is removed, iron disassembles by emitting heat radiation, and the original disordered state is restored, meaning that iron, especially soft iron, loses its magnetism. The steel, on the other hand, partially retains its order and becomes a permanent magnet.

The locations of the magnets, where the magnetic force is the strongest is called the magnetic pole, have the same force applied to each pole. Uniform magnetic poles repel, different poles attract each other. The magnetic field formed around the magnets can be sensed by iron filings, which, by division, act as tiny magnets.

The substantive explanation (and examination) of magnetism began when, in 1820, Hans Christian OERSTED showed that electric current (moving charges) had a magnetic field. This was followed by André-Marie AMPÉRE's assumption that magnetic materials contain molecular circular currents whose ordering is caused by macroscopic magnetism. Subsequently, in the knowledge of the Bohr atomic model, it was assumed that these circular currents are created by electrons orbiting the nucleus. However, this explanation based on the orbital motion of the electrons did not match the experience. To our knowledge, the role of the magnetic field generated by the electron spin (electron spin momentum), discovered much later, is decisive in explaining magnetism.

Comment:

Some nuclei also have magnetic effects that are orders of magnitude smaller than electrons.

3.3.2. Characterization of the magnetics field

3.3.2.1. Magnetic induction

Experience shows that the magnetic field acts only on moving charges (the electric field also acts on resting and moving charges). This force is always perpendicular to the charge velocity vector and its maximum is proportional to the charge Q and the velocity v:

$$F_{\max} \sim Qv$$

or

$$F_{\max} = QvB$$

where the proportional coefficient B is the vector quantity at a given point in the magnetic field called magnetic induction $\vec{B}$ and its SI unit is $\frac{N}{C \cdot \frac{m}{s}} = \frac{V \cdot s}{m^2} = T$ (tesla).

The magnitude of the magnetic induction on the Earth's surface is 30 to 60 μT, whereas in MRI devices, superconducting coils are usually used to generate 1 to 10 T magnetic fields.

The equation defining the induction thus contains three vector quantities ($\vec{F}$, $\vec{v}$, $\vec{B}$). This is only possible if it is a vector product of $\vec{v}$ and $\vec{B}$, and this results in a vector:

$$\boxed{\vec{F}_L = Q\vec{v} \times \vec{B}}.$$

This force is called the magnetic Lorentz force. The direction of $\vec{B}$, by convention, with a positive Q charge, is such that $\vec{v}$, $\vec{B}$, and $\vec{F}$ follow each other as the thumb, forefinger, and middle finger of our right hand.

Of which size of $\vec{F}_L$ is:

$$F_L = QvB \sin a$$

where α is the angle enclosed by $\vec{v}$ and $\vec{B}$. The figures show some magnetic fields with induction lines (Figure 3.21: magnetic rod; 3.22: straight conductor; 3.23: solenoid coil; 3.24: circular current; 3.25: parallel conductors)

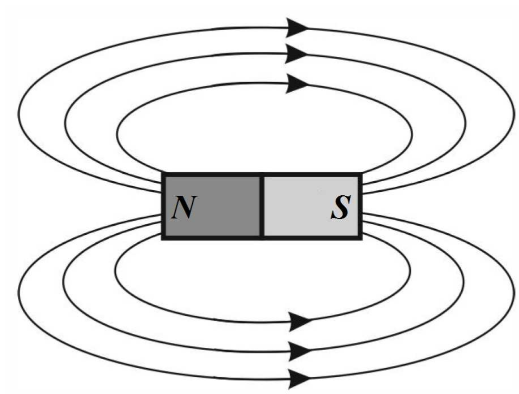

Figure 3.21

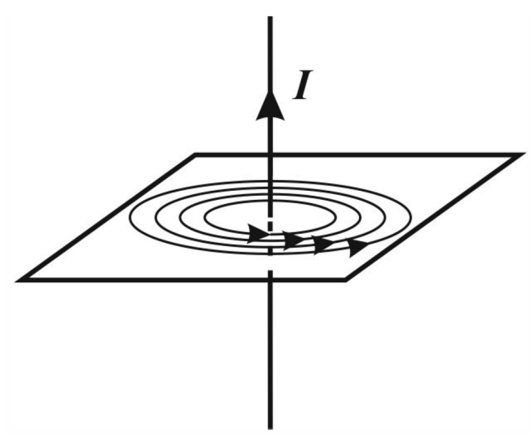

Figure 3.22

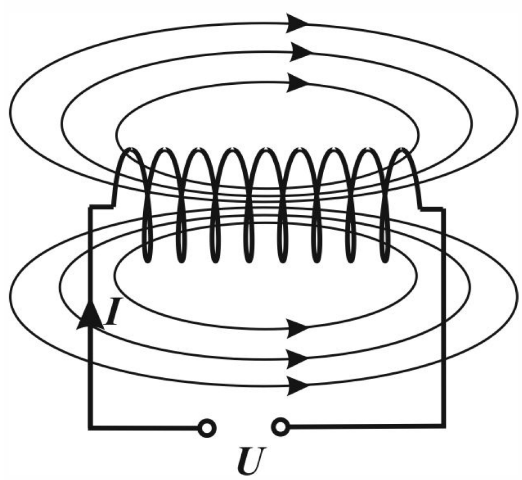

Figure 3.23

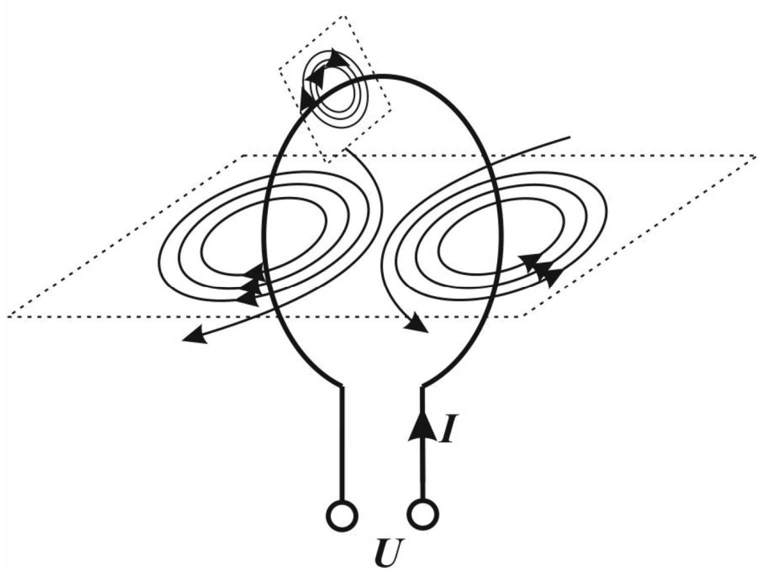

Figure 3.24

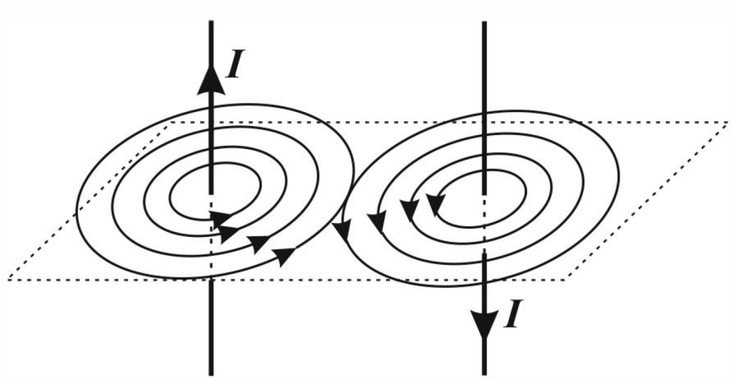

Figure 3.25

3.3.2.2. The flux of the magnetic field

Magnetic flux is interpreted in the same way as electric flux. The flux Φ of a homogeneous magnetic field B induced perpendicular to plane A is the product

$$\boxed{\Phi = BA}.$$

If $\vec{B}$ encloses angle α with $\vec{n}$ normal vector to planar surface A, then the flux is the product of the normal component of $\vec{B}$ to the surface and the size of the surface,

$$\Phi = BA\cos\alpha = \vec{B}\cdot\vec{A}$$

It is scalar product in which $\vec{A} = A\vec{n}$, the surface vector. The numerical value of the flux shows the number of magnetic induction lines passing through surface A. Its SI unit is $T \cdot m^2 = V \cdot s = Wb$ (weber).

In an inhomogeneous magnetic field:

$$\Phi = \int_A \vec{B}\cdot d\vec{A}.$$

Since there are no magnetic monopoles, the magnetic flux, or source power, of any closed surface in the magnetic field is zero:

$$\boxed{\oint \vec{B}\cdot d\vec{A} = 0}.$$

The above law, or the magnetic Gauss Law, is the 3rd equation of Maxwell's united electromagnetic theory. It describes the flux, or source power, of a static magnetic field under vacuum. The law, in words:

"The static magnetic field is source-free, and the closed surface enters as many induction lines as it exits."

3.3.3. Force effects in magnetic field

3.3.3.1. Force on a point charge

The magnetic field induced of $\vec{B}$ exerts force only on a moving charge if the velocity $\vec{v}$ of the charge Q has a component perpendicular to the direction of $\vec{B}$. The charge moving parallel to $\vec{B}$ is not affected by magnetic forces.

If the magnetic induction $\vec{B}$ is perpendicular to the charge velocity $\vec{v}$, then the force $\vec{F}_L$ acts perpendicular to both $\vec{B}$ and $\vec{v}$ and can change only the direction of the charge velocity, not its magnitude. The Mechanics c. we have seen that a force perpendicular to the velocity of the body forces the body to orbit at any moment. The centripetal force required for smooth circular motion in this case is provided by the Lorentz force. Applying the basic equation of Dynamics:

$$\sum \vec{F} = m\vec{a},$$

from where

$$QvB = m\frac{v^2}{r}$$

of which the radius of the circle is:

$$r = \frac{mv}{QB}.$$

Time required to run a complete circle (period):

$$T = \frac{2\pi r}{v} = 2\pi\frac{m}{QB},$$

that is, independent of the velocity of the point charge.

Comment:

If the magnetic induction of $\vec{B}$ *is not perpendicular to the velocity* $\vec{v}$ *of the charge, then* $\vec{B}$ *exerts a Lorentz force only on the perpendicular component of the velocity, its parallel component does not change, i.e., the charge moves smoothly in the direction of* $\vec{B}$*. The resulting motion path will be a helix of even thread pitch.*

Devices and phenomena based on the force of the magnetic field on moving charges:

- electron microscopes "Magnetic lenses",
- cyclotron particle accelerator,
- separation of particles (ions) of the same charge but of different mass in mass spectroscopes,
- deflection of electrons in cathode ray tubes,
- the helical movement of electrically charged particles of the "solar wind", deflected by the Earth's magnetic field, towards the poles, causing the phenomenon of aurora borealis.

3.3.3.2. Torque acting on a magnetic dipole in a magnetic field

In nature, magnetic monopoles do not exist, every magnet is a dipole. Like the electric dipole moment of the electric dipole, we introduce the magnetic dipole momentum. The magnetic moment of a dipole with a pole strength $+p$(N) and $-p$(S) at a distance l is the product

$$\boxed{\vec{m} = p\vec{l}}.$$

By convention, its direction is the same as the vector $\vec{l}$ from $-p$ to $+p$. The magnetic dipole consisting of $+p$ and $-p$ poles is called a Coulomb dipole.

The conductor frame A, which is passed through a current of I strength and consists of N threads, also acts as a magnet, the so-called Ampere dipole with magnitude of magnetic moment:

$$\boxed{m = INA}.$$

The homogeneous static magnetic field, like the $\vec{F}_e = Q\vec{E}$ electric force applied to the Q charge in the electrostatic field, exerts a force pair to the magnetic dipole:

$$\vec{F}_m^- = -p\vec{B} \text{ and } \vec{F}_m^+ = +p\vec{B}$$

So it applies torque when the action line of the force pair does not coincide with the axis of the dipole. The magnitude of this is given by writing the torque to the $-p$ pole as the axis of rotation:

$$M = Fk = pBL\sin\alpha = mB\sin\alpha,$$

where α is the angle enclosed by the axis of the magnetic dipole and the induction lines of the magnetic field. Its direction follows the vectorial product:

$$\boxed{\vec{M} = \vec{m} \times \vec{B}}.$$

As a result of the torque, the magnetic dipole rotates in the direction of magnetic induction. Meanwhile, the magnetic field performs work on the dipole during rotation. By convention, the potential energy E_p of a dipole with a magnetic momentum m is defined as the work done by a magnetic field of induction $\vec{B}$ to rotate it from an angle of $\alpha_0 = 90°$ to any angle α. How to calculate this:

$$E_p = W = \int_{\frac{\pi}{2}}^{\alpha} M d\alpha = \int_{\frac{\pi}{2}}^{\alpha} mB\sin\alpha d\alpha = -mB\cos\alpha = -\vec{m} \cdot \vec{B}$$

Comments:

1. The work of torque M, $W = \int M d\alpha$ was used in an analogous manner to the work of force F, $W = \int F ds$, without derivation.

2. The smallest potential energy of the dipole is at $\alpha = 0^o$, then $E_p = -mB$. This is a stable equilibrium position because the magnitude of the forces and torque applied to the dipole is zero. In this case, the magnetic momentum vector m of the magnetic dipole points parallel to the external magnetic field in the same direction, so called parallel setting.

3. The maximum potential energy of the dipole is in α = 180° position, then $E_p = mB$. This is an unstable equilibrium position because the magnitude of the forces and torque applied to the dipole is zero, but it shifts from its equilibrium position to the smallest effect and moves to a stable equilibrium position. In this case, the magnetic moment vector m of the magnetic dipole points in the opposite direction to the external magnetic field, so-called antiparallel setting.

Magnetic resonance imaging (MRI) of nuclei investigates the behavior of nuclei with magnetic moment in an external magnetic field.

3.3.3.3. Force between parallel conductors (Ampere's law)

Examining the force effect between two conductors of infinite length, of negligible cross-section, with currents I_1 and I_2, in the arrangement of Figure 3.26 we find that, for currents of the same direction, an attractive, for currents in the opposite direction a repulsive force occurs.

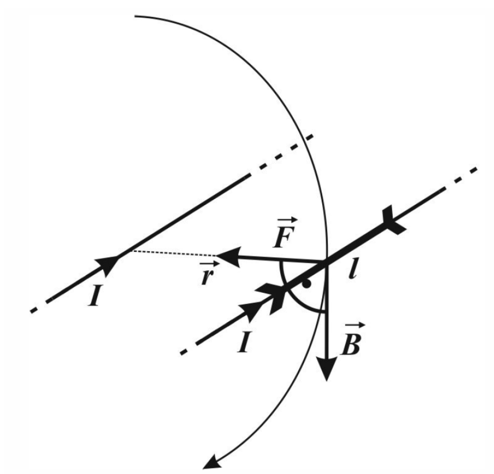

Figure 3.26

It is well known that the magnetic field formed around a straight current conductor is cylindrical symmetrically swirling, so that the induction of the magnetic field formed around the conductor 1 is perpendicular to the conductor 2. If the currents I_1 and I_2 flow in the same direction and there is an attractive force between them, it follows that the direction of the magnetic field induction of conductor 1 is calculated by the right-hand rule. Negative charges in line 2 are then affected by the force pointing to line 1. In the opposite current direction, the force points to the opposite direction.

Comment:

The definition of amperage was based on this force in 1946. With this, SI opted a technically easier and more accurate current measurement instead of a more basic physical amount, the electric charge.

3.3.4. Magnetic properties of materials

The induction B_0 of the magnetic field in vacuum generated by the conduction currents changes to B if the induction lines of the magnetic field are in some material. This change is due to the fact that the own magnetic field of the molecular magnetizing currents that make up the material is superimposed on the external magnetic field. The effect of the medium on modifying the external magnetic field is characterized by the quotient of the two inductions and is called the relative permeability:

$$\mu_r = \frac{B}{B_0}$$

It follows from the definition that $\mu_{r,vacuum}$=1. Based on the μ_r value, materials can be grouped magnetically as:

- Diamagnetic materials: μ_r<1 (eg water, copper)
- Paramagnetic materials: μ_r>1 (eg aluminium, air)
- Ferromagnetic materials: $\mu_r \gg 1$ (eg iron and its alloys, nickel), the relative permeability of these materials is strongly dependent on external induction and temperature. Above a critical temperature (Curie point) the ferromagnetic property disappears and the material becomes paramagnetic.

3.3.5. Amper's excitation law

Ampere recognized that the summation (integral) of the magnetic induction $\vec{B}$ for a closed curve is proportional to the sum of the amperage of the conduction currents flowing through the surface bounded by the closed curve (Figure 3.27):

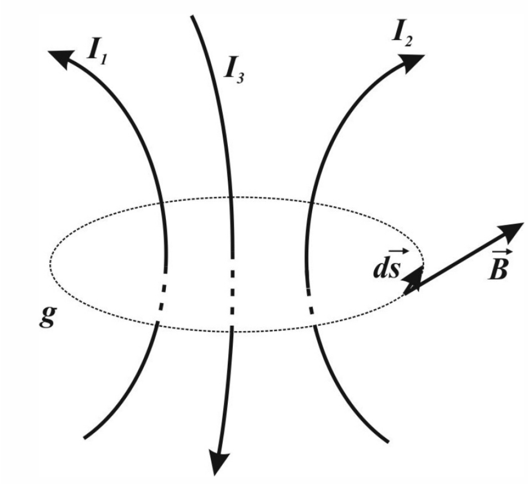

Figure 3.27

Mathematically:

$$\oint_g \vec{B} \cdot d\vec{s} \sim \sum I$$

(in our figure $\sum I = I_1 + I_2 + I_3$) and the proportionality factor to be introduced is the permeability $\mu = \mu_0 \mu_r$, where $\mu_0 = \frac{1}{\varepsilon_0 c} = 4\pi \cdot 10^{-7} \frac{V \cdot s}{A \cdot m}$ is the permeability of the vacuum, so:

$$\oint_g \vec{B} \cdot d\vec{s} = \mu \sum I$$

In general, if the charge current through surface A is not constant, then the right-hand side of the relation is the integral of the current density vector J for surface A:

$$\boxed{\oint_g \vec{B} \cdot d\vec{s} = \mu \int_A \vec{J} \cdot d\vec{A}}.$$

This is Ampere's law of excitation. The calculation of the integral of the closed curve is presented only in very special (symmetric) cases in connection with examples.

Example: Magnetic field of a cylindrical coil (solenoid) inside the coil

Determine the magnetic induction of a cylindrical coil (solenoid) of length l and N turns passed by a current of amperage I inside the coil.

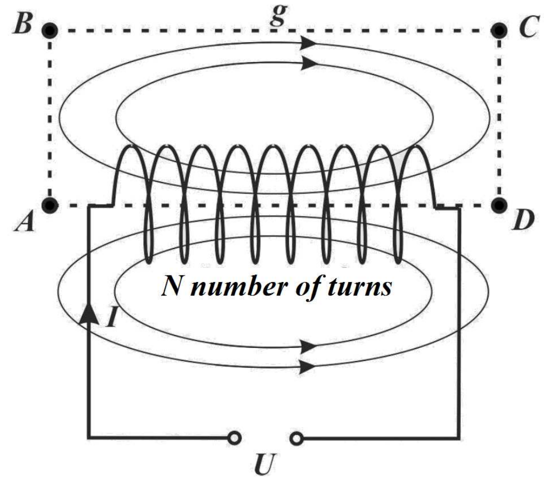

Figure 3.28

Solution:

According to Figure 3.28, based on the excitation law applied to the closed loop defined by the *ABCD* points:

$$\oint_g \vec{B} \cdot d\vec{s} = \int_A^B \vec{B} \cdot d\vec{s} + \int_B^C \vec{B} \cdot d\vec{s} + \int_C^D \vec{B} \cdot d\vec{s} + \int_D^A \vec{B} \cdot d\vec{s}\,,$$

and since in the out-of-coil space (sections *AB*, *BC* and *CD*) the line integration of the induction is negligibly small compared to its internal space, using Ampere's excitation law:

$$\oint_g \vec{B} \cdot d\vec{s} = \int_D^A \vec{B} \cdot d\vec{s} = B\int_D^A ds = Bl = \mu IN\,.$$

Of which B can be expressed:

$$B = \mu \frac{NI}{l}\,,$$

thus, the induction inside the coil is homogeneous with high accuracy.

Note: MRI equipment provides a homogeneous magnetic field with such a coil.

3.3.6. Time constant electric and magnetic field (summary)

3.3.6.1. Characterization of fields

A common feature of the discussed electric and magnetic fields is that the vector functions $\vec{E}$ and $\vec{B}$ characterizing the fields are independent of time t, static, and the lines of force representing them are also constant in time (they do not "move"). The basic reason for the formation of both fields can be traced back to the electric charge: static charges create electric, moving charges (currents) create electric and magnetic fields.

Static fields can be considered independent of each other, this is clearly seen in the spatial equations describing them.

3.3.6.1.1. The static electric field

1. Source strength:

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{\sum Q}{\varepsilon_0}}$$

(has source)

2. Vortex strength:

$$\boxed{\oint \vec{E} \cdot d\vec{s} = 0}$$

(vortex-free)

3.3.6.1.1. The static magnetic field

1. Source strength:

$$\boxed{\oint \vec{B} \cdot d\vec{A} = 0}$$

(source-free)

2. Vortex strength:

$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu \int \vec{J} \cdot d\vec{A}}$$

(has vortex)

The left side of the space equations describing the source strength expresses the flux calculated for the electrical and magnetic closed surface. The static electric field has a source, the sources are electric charges. The static magnetic field, on the other hand, is source-free, with no real magnetic "charges". Time as a variable does not occur in any of the four equations. Neither equation contains $\vec{E}$ and $\vec{B}$ at the same time. We will see that the relationship between electric and magnetic fields only occurs if the fields change over time (non-static).

3.3.6.2. Characterization of forces

We also dealt with the force of the field on the Q charge in electrostatics and magnetostatics:

3.3.6.2.1. Electric force

The electric field with field strength $\vec{E}$ exerts an electric force in the same direction as the field strength for the $+Q$ point charge in it, and in the opposite direction for the $-Q$ point charge:

$$\boxed{\vec{F}_e = Q\vec{E}}.$$

3.3.6.2.2. Magnetic Lorentz force

For a point Q charge moving at a velocity $\vec{v}$ in a magnetic field with induction $\vec{B}$, the magnetic field exerts a Lorentz force perpendicular to the velocity and magnetic induction in the right-hand direction:

$$\boxed{\vec{F}_L = Q\vec{v} \times \vec{B}}.$$

3.3.6.2.3. Electric and magnetic field

If the charge Q moving at speed $\vec{v}$ moves simultaneously in an electric and magnetic field, the magnitude of the total Lorentz electromagnetic force is:

$$\boxed{\vec{F} = \vec{F}_e + \vec{F}_L = Q\vec{E} + Q\vec{v} \times \vec{B} = Q\left(\vec{E} + \vec{v} \times \vec{B}\right)}.$$

Since the magnetic Lorentz force is always perpendicular to the velocity and the magnetic induction, it does not perform work, the work of the electric force is different from zero in the case of a displacement parallel to the electric field strength.

3.4. Time-varying electromagnetic field

After Oersted's discovery of the magnetic boundary of current, many thought that this relationship between electricity and magnetism should also be a non-one-way, reverse effect, i.e., an electric field could be used to generate (induce) an electric field.

Shortly after Oersted's discovery, Michael FARADAY discovered this "reverse" phenomenon, electromagnetic induction. It turned out that if the electric resp. the magnetic field changes over time, then the two fields are present together in the process, their effects cannot be separated.

3.4.1. Rest induction

Using the experiment of Figure 3.29, it can be shown that a magnetic field can be used to generate current in the resting coil. If the permanent bar magnet is at rest relative to the conducting loop, the ammeter will not indicate electrical current (I=0).

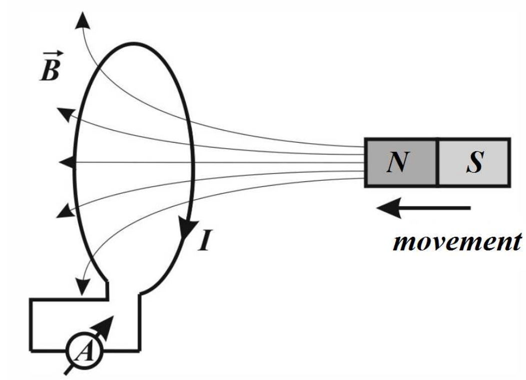

Figure 3.29

When the rod magnet is approached to the conductor loop, the ammeter measures a current of $I > 0$. A repulsive effect on the rod magnet is observed (Figure 3.29). If the rod magnet is removed from the conducting loop, the ammeter $I < 0$ indicates a current in the opposite direction. We find an attractive effect on the rod magnet in this case (Figure 3.30).

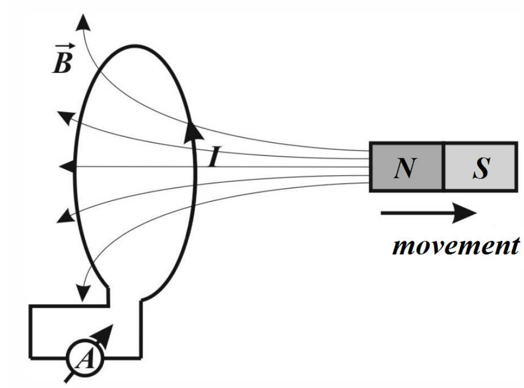

Figure 3.30

The phenomenon observed in the above experiments can be explained by the fact that by moving the magnet, a magnetic field of variable flux (not constant in time) in the conducting loop induces an electric field around itself, the lines of force of which close to themselves and are characterized by induced voltage. Under the influence of a closed circuit the so-called. induced electric current is generated.

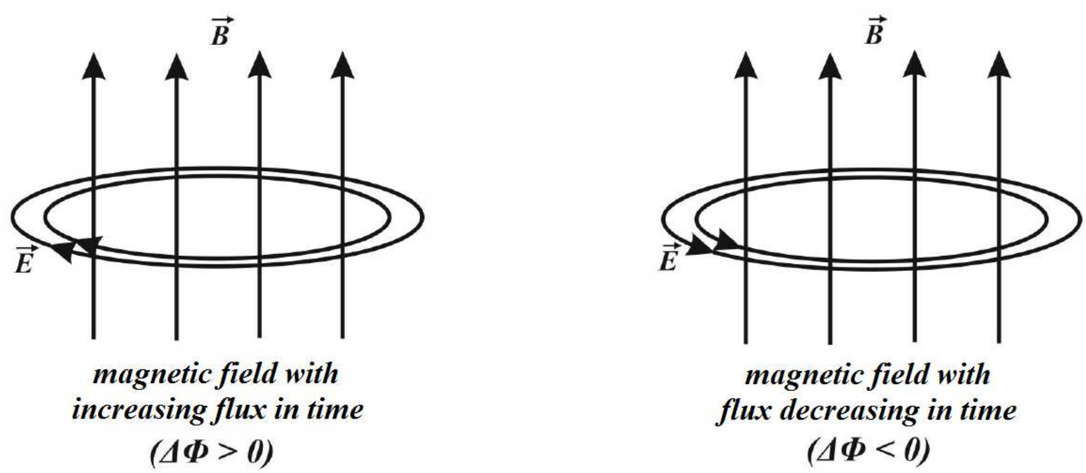

Figure 3.31

The induced electric field creating the motion of the charges is characterized by the instantaneous voltage, which is proportional to the change in the magnetic flux Φ per unit time Δt:

$$U_i = -\frac{\Delta\Phi}{\Delta t},$$

for $\Delta t \to 0$, Faraday's law of induction, respectively:

$$U_i = -\frac{d\Phi}{dt}.$$

The negative sign refers to the so-called Lenz law:

"The current induced in the conducting loop is always in such a direction that its magnetic field prevents the change that creates the rest induction."

The Lenz law is a consequence of the principle of conservation of energy. If e.g. in the experiment of Fig. 3.29 (magnet moving towards the conducting loop) the direction of the induced current would be opposite, it would attract a rod magnet approaching the conductor and thus gain mechanical work from “nothing” in addition to the Joule heat.

Comment: If a coil with N turns is used instead of a conducting loop, the voltages induced in each turn will add up due to the series connection of each turn and thus:

$$U_i = -N\frac{d\Phi}{dt}.$$

Since the induced voltage can be expressed by the induced electric field strength $\vec{E}$:

$$U_i = \int_A^B \vec{E}\cdot d\vec{s},$$

therefore, by summing on a closed curve:

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi}{dt}.$$

On the left side of the equation, starting from the definition of magnetic flux Φ=BA, the flux $d\Phi$ per dA surface element is:

$$d\Phi = BdA,$$

therefore, the total flux per surface A integrates this, and so the equation:

$$\boxed{\oint \vec{E}\cdot d\vec{s} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A}}.$$

Comment: The symbol ∂ is used to denote a derivative of a single variable of a quantity dependent on several variables, the so-called partial derivative.

The above law, or Faraday–Maxwell’s law of induction, is described in Maxwell’s Unified Electromagnetic Theory II. equation, which describes the vortex strength of an electric field induced by a time-varying magnetic field in a vacuum. The law is worded:

“The electric field induced by the time-varying magnetic field has vortex, the force lines are self-closing curves.”

Comments:

1. At rest induction, the change in flux is always caused by the change in the magnetic field over time, this was achieved in the experiment by moving the magnetic rod.

2. The induced electric field is not bound to the charges present, it is source-free:

$$\oint \vec{E} \cdot d\vec{A} = 0 .$$

3. At rest induction, the conductor has only an indicator role, the time-varying magnetic field is surrounded by an electric field even in a vacuum.

An electron accelerator called Betatron takes advantage of the phenomenon that a magnetic field forces charges into orbit, and an electric field induced by a time-varying magnetic field also accelerates them along the orbit. High kinetic energy electron beams can be used, for example, to produce X-rays for therapeutic purposes.

3.4.2. Self-induction

Self-induction is a special case of quiescent induction, where in a closed conductor (e.g. in a coil) a changing magnetic field due to a change in current over time induces a voltage in the conductor itself. Using the result obtained for the magnetic induction according to the above example for the instantaneous voltage induced in the coil, based on the above:

$$U_i = -N\frac{d\Phi}{dt} = -N\frac{d(BA)}{dt} = -NA\frac{dB}{dt} = -NA\frac{d}{dt}\left(\mu\frac{Ni}{l}\right) = -NA\mu\frac{N}{l}\frac{di}{dt} = -\mu\frac{N^2A}{l}\cdot\frac{di}{dt},$$

where $\mu = \mu_0\mu_r$ is the permeability of the material in the coil and $\mu\frac{N^2A}{l} = L$ is the coefficient characteristic of the coil, its name is self-induction coefficient (inductance of the coil), its unit in SI is $\frac{V \cdot s}{A} = H$ (henry). So:

$$\boxed{U_i = -L\frac{di}{dt}}.$$

The inductance L is the same characteristic of the coil as the capacity C of the capacitor. It can be seen that the inductance is strongly influenced by the relative magnetic permeability μ_r of the material filling the coil (e.g. iron core) and the number of turns N of the coil.

3.4.2.1. Energy and energy density stored in the magnetic field of the coil

Like the energy stored in the electric field of the capacitor, energy is stored in the magnetic field of the coil passed by the current. Without conduction, the energy of the induction magnetic field B generated inside the coil with inductance L under the influence of a direct current of amperage I:

$$\boxed{E_L = \frac{1}{2} L I^2},$$

and its energy density is:

$$w_L = \frac{E_L}{V} = \frac{1}{2}\frac{B^2}{\mu}.$$

3.4.3. Mutual induction

Mutual induction occurs in the case of coils connected by a common flux, which can be provided according to Figure 3.32.

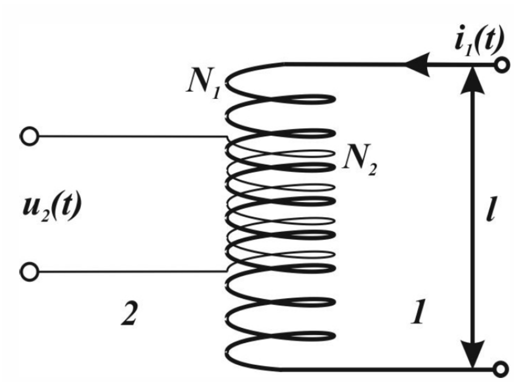

Figure 3.32

Due to the change in the instantaneous induction magnetic field

$$B_1 = \mu \frac{i_1 N}{l}$$

of the instantaneous current i_1 in the coil 1, the instantaneous value of the induced voltage generated in the coil 2 in the case of a common magnetic flux:

$$u_2 = -N_2 \frac{d\Phi_2}{dt} = -N_2 \frac{d(BA)}{dt} = -N_2 A \frac{dB}{dt} = -N_2 A \mu \frac{N_1}{l} \frac{di_1}{dt}$$

where the quotient $\mu \frac{N_1 N_2}{l} = M$ is a constant characteristic of the common geometrical conditions of the two coils, called the mutual induction coefficient, its unit in SI is $\frac{V \cdot s}{A} = H$ (henry). So:

$$u_2(t) = -M \frac{di_1}{dt}.$$

The value of M is a value that characterizes the degree of coupling between the two coils. Transformers operate on the principle of mutual induction.

3.4.3.1. The transformer

The transformer consists of two coils wound on a common iron core. One is supplied with alternating current (usually connected to a sinusoidal alternating voltage generator), this is the primary winding with N_1 turns. The voltage induced in the other secondary winding with N_2 turns is obtained (Figure 3.33). The common iron core ensures coupling and the same magnetic flux.

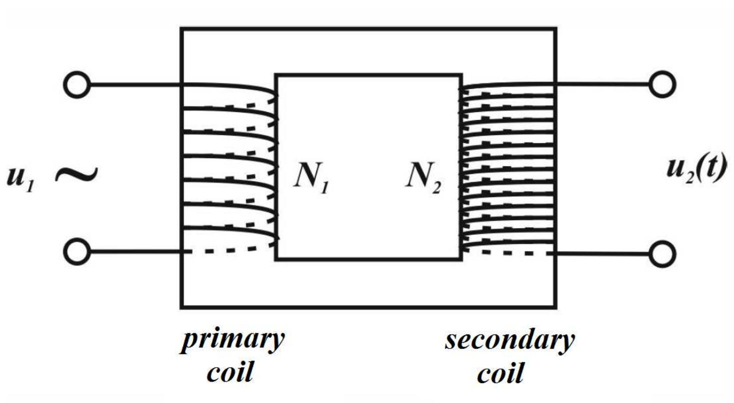

Figure 3.33

3.4.3.1.1. Idle condition

The idle state of the transformer is considered when there is no consumer on the secondary side ($i_2 = 0$). Then the magnetic field excited by the current i_1 of the voltage generator u_1 connected to the primary coil also passes inside the secondary coil. Thus, the voltage per turn of the primary coil is equal to the voltage induced per turn of the secondary coil. This means that the "gear ratio" of the transformer:

$$\frac{N_2}{N_1} = \frac{u_2}{u_1}$$

In the case of $N_2>N_1$ we speak of up-transformation, in the case of $N_2<N_1$ we speak of down-transformation, and in the case of $N_2=N_1$ we speak of isolating transformation.

3.4.3.1.2. Loaded condition

We speak of a loaded operating state if there is a consumer with resistance R on the secondary side, then a current of i_2 current flows through the consumer. The efficiency of a well-designed and constructed transformer is close to 100%, which means that the power P_1 absorbed on the primary side is almost entirely transmitted on the secondary side ($P_2 \approx P_1$), i.e.:

$$u_1 i_1 \approx u_2 i_2 .$$

In the case of up-transformation ($u_2>u_1$) the strength of the secondary side current is smaller, while in the case of down-transformation ($u_2<u_1$) the strength of the secondary side current is higher than the strength of the primary side current.

The voltage of high-voltage (120 to 750 kV) transmission lines is transformed (usually in several steps) to reach 230 V for residential consumers. The voltage generated by the power plants is transformed to allow the lowest possible current to flow in the transmission line, which is the cause of Joule's heat loss.

3.4.4. Motion induction

We have seen above that a change in the flux Φ of a magnetic field creates an electric field characterized by an electric field strength $\vec{E}$ and an induced voltage U_i in cases where the change in flux is caused by a change in the number of magnetic induction lines passing through a given surface A (rest induction).

In the following, we consider the case of the change of the magnetic flux Φ when the surface A of the area bounded by a conductor placed in a magnetic field with a constant induction $\vec{B}$ changes (Figure 3.34).

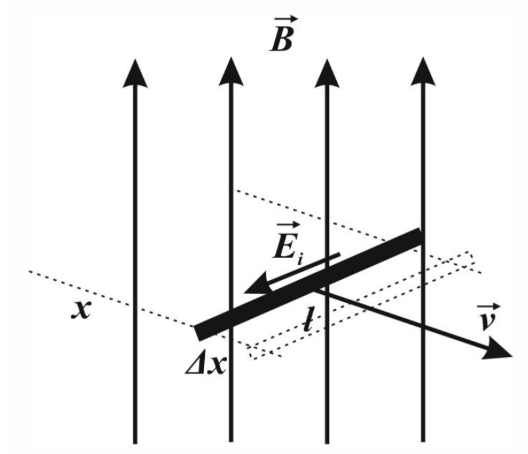

Figure 3.34

The movable line of length l in the arrangement according to Fig. 3.35 is connected to a consumer with resistance R.

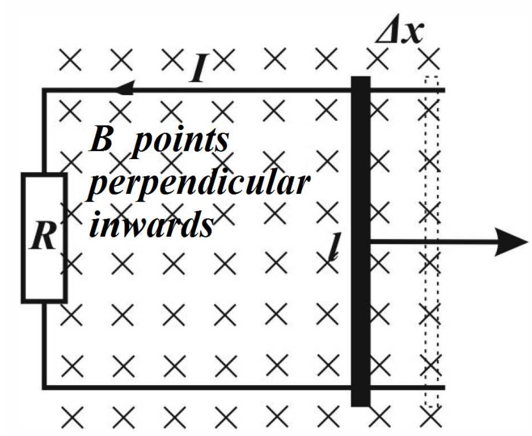

Figure 3.35

The magnetic field exerts a magnetic Lorentz force on the free electrons with a charge $-e$ moving together with the conductor moving at speed v:

$$\vec{F}_L = -e\vec{v} \times \vec{B}\,.$$

As a result, the charges in the conductor are separated: free electrons accumulate at one end of the conductor, resulting in a positive excess charge at the other end of the conductor. Induced voltage perpendicular conditions ($\vec{v}$, $\vec{B}$ and $\vec{l}$) characterising the charge-separating effect:

$$U_i = -\frac{d\Phi}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -B\frac{d(lx)}{dt} = -Bl\frac{dx}{dt} = -Blv\,.$$

At the same time, the voltage characteristic of the electric field of the separated charges:

$$U = -U_i = Blv\,.$$

In a closed circuit containing a consumer with resistance R, this voltage U generates a so-called induced electric current of amperage I, which according to Ohm's law:

$$I = \frac{U}{R} = \frac{Blv}{R}\,.$$

This current is always in such a direction as to impede the motion that creates the induction (Lenz's law).

Thus, during motion induction, we generate electrical energy through mechanical work. This is the physical basis for the operation of electromechanical generators (power generators).

3.4.4.1. Production of alternating voltage and alternating current

To produce the alternating voltage, the conductor frame (or coil) is rotated at $\omega = constant$ angular velocity in a magnetic field perpendicular to the axis of rotation (Figure 3.36).

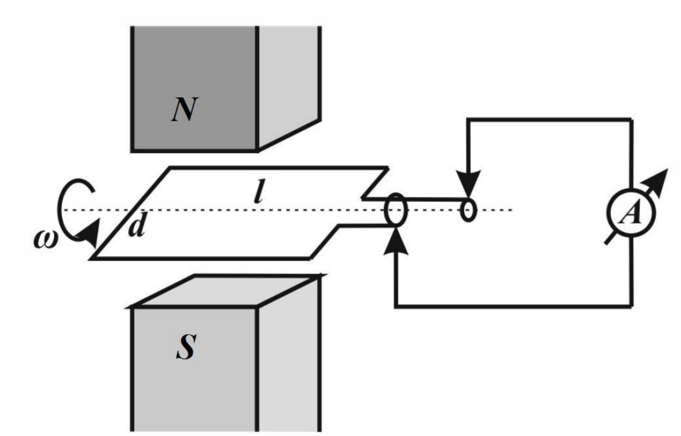

Figure 3.36

The component of the speed of the l side of the conducting frame perpendicular to the magnetic field as shown in Figure 3.37:

$$v_n = v \sin\alpha = \frac{d}{2}\omega \sin\omega t\,,$$

and therefore the instantaneous induced terminal voltage between the ends of its side of length l perpendicular to the magnetic field:

$$u = Blv_n = Bl\frac{d}{2}\omega \sin\omega t\,.$$

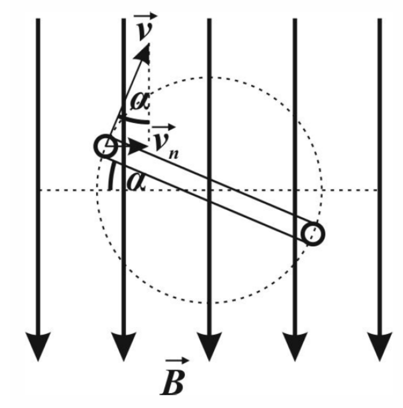

Figure 3.37

If the section of length l on the opposite side is also taken into account, due to the serial connection, on the terminals of the conducting frame

$$u = 2Bl\frac{d}{2}\omega \sin \omega t = Bld\omega \sin \omega t = BA\omega \sin \omega t$$

an instantaneous, sinusoidally varying value of induced terminal voltage occurs. Since $BA\omega = constant$, the instantaneous voltage u has a peak value U_0, therefore:

$$\boxed{u = U_0 \sin \omega t}.$$

If a consumer with resistance R is connected to the terminals of the conducting frame, then in the circuit according to Ohm's law

$$i = \frac{u}{R} = \frac{U_0}{R}\sin \omega t = I_0 \sin \omega t$$

instantaneous and I_0 peak AC current is flowing. Thus, in the case of $\omega = constant$, a purely sinusoidal voltage, resp. electric current can be generated. Using N-turn coils due to serial connection of threads

$$U_0 = NBA\omega$$

maximum voltage (peak voltage) can be generated.

3.4.4.2. Vortex current

Induced electrical current can be generated not only in the conductor circuit built for this purpose, but also in extensive metal bodies (e.g., the iron core of the coil) through induction. As a result of the induced voltage characteristic of the vortex-induced electric field, an electric current, the so-called vortex current, flows in the metal body (Figure 3.38):

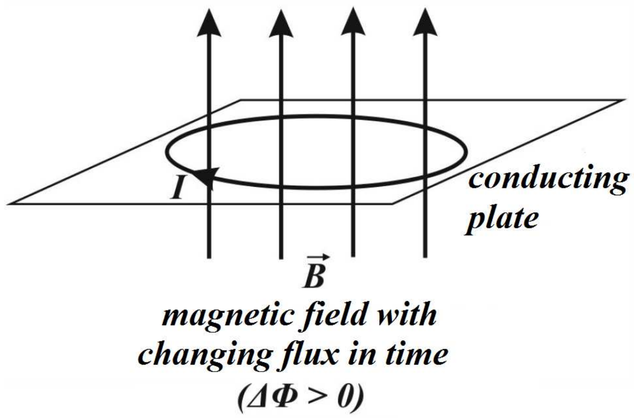

Figure 3.38

This is usually harmful because the heat it generates causes energy loss, but e.g. this is exactly what is used in so-called induction furnaces to melt metals. Where we want to reduce the loss, the iron cores in the magnetic field are plated and insulated from each other with a layer of lacquer. At high frequencies (not at speeds typical of mechanical machines) the so-called powder ferrite core produced by powder metallurgy.

3.4.4.3. Effective current and effective voltage

Because the values of alternating currents and voltages vary from moment to moment, it is almost impossible to measure the instantaneous values (at most it can be illustrated with an oscilloscope) and this is usually not necessary. Instead, we use the effective value of alternating current and voltage.

By the effective value of alternating current I_{eff} is meant the amperage of the direct current that generates the same amount of Q Joule-heat in a consumer of resistance R over the same amount of time as the alternating current. The alternating current of instantaneous strength i in the consumer with resistance R during time dt developes

$$dQ = Ri^2 dt$$

heat, therefore the heat developed during the whole period T:

$$Q = \int_0^T Ri^2 dt\,.$$

At the same time, a direct current of amperage I_{eff} in the same consumer of resistance R for the same amount of time T developes:

$$Q = RI_{eff}^2 T$$

heat, and by definition this is equal to the heat produced by alternating current:

$$RI_{eff}^2 T = \int_0^T Ri^2 dt .$$

From this, the effective value can be calculated:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} .$$

For sinusoidal *i* momentary current after integration:

$$\boxed{I_{eff} = \frac{I_0}{\sqrt{2}}}$$

is given, where I_0 is the peak current. And the effective voltage after the application of Ohm's law:

$$\boxed{U_{eff} = \frac{U_0}{\sqrt{2}}}$$

The peak value of the residential mains voltage $U_{eff} = 230V$, eg:

$$U_0 = U_{eff}\sqrt{2} \approx 325V .$$

3.4.5. AC circuits

We have seen above how sinusoidal alternating electrical voltages can be used. generate electricity electromechanically. Their current values are:

$$u = U_0 \sin \omega t$$

and

$$i = I_0 \sin \omega t ,$$

their effective values are:

$$U_{eff} = \frac{U_0}{\sqrt{2}}$$

and

$$I_{eff} = \frac{I_0}{\sqrt{2}} ,$$

where U_0 and I_0 are the peak values, ω is the circular frequency (numerically equal to the angular velocity of the rotating coil). Also: $\omega = \frac{2\pi}{T} = 2\pi f$, where f is the frequency and T is the period time.

With this method it is possible to produce a voltage or current with a frequency of a few hundred Hz, the production of higher frequencies with electromechanical devices is not possible due to technical reasons. The voltage and current used in the network have a frequency of 50 *Hz* according to the European standard, which can be achieved with a generator speed of $3000\ \frac{1}{\text{min}}$.

Comment:

The USA and Japan use a standard with a mains frequency of f = 60 Hz and an effective voltage of $U_{eff} = 110V$.

Much higher frequency electrical voltage and electrical current can only be achieved with electronic devices without vibrating components, vibrators (see later).

In a conductor carrying a purely sinusoidal current, the charge carriers (electrons) perform a harmonic oscillating motion at a circular frequency ω, so the alternating current must be treated as a harmonic oscillating motion.

3.4.5.1. Circuit elements in alternating current networks

3.4.5.1.1. Consumer with ohmic resistance

In the AC circuit of Figure 3.39, the instantaneous voltage per consumer *R* is:

$$u(t) = U_0 \sin \omega t$$

and the instantaneous current is

$$i(t) = I_0 \sin \omega t\ .$$

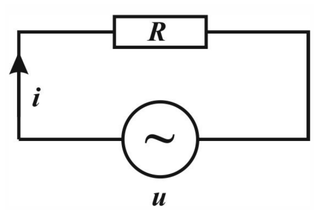

Figure 3.39

Thus, the voltage and the electric current vibrate in the same phase, their phase difference, the so-called ohmic phase angle is:

$$\Delta\varphi_R = \omega t - \omega t = 0\ ,$$

the current alternates “synchronously” with the voltage. Resistor *R* converts the electrical energy *UIt* taken from the mains entirely into Joule heat.

3.4.5.1.2. Consumer with inductive resistance

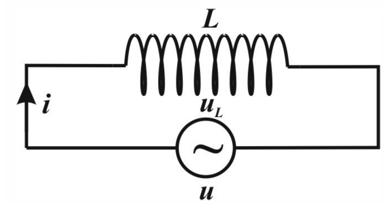

Figure 3.40

In the circuit containing the self-induction coil (Figure 3.40), the magnitude of the instantaneous voltage u of the power source according to Kirchhoff's II law is the same as the u_L induced in the coil:

$$u = u_L ,$$

and by

$$i(t) = I_0 \sin \omega t$$

instantaneous current using the law of self-induction:

$$u = L\frac{di(t)}{dt} = L\frac{d}{dt}(I_0 \sin \omega t) = L\omega I_0 \cos \omega t = L\omega I_0 \sin\left(\omega t + \frac{\pi}{2}\right).$$

Then by introducing $U_0 = L\omega I_0$:

$$u(t) = U_0 \sin\left(\omega t + \frac{\pi}{2}\right).$$

From the relation $U_0 = L\omega I_0$ we introduce the analogy of Ohm's law with

$$\boxed{X_L = \omega L}$$

inductive resistance (inductive reactance). Difference between phase angles of voltage and current:

$$\Delta\varphi_L = \left(\omega t + \frac{\pi}{2}\right) - \omega t = \frac{\pi}{2}.$$

Based on this, it can be concluded that in the case of an inductive consumer, the phase of the current is delayed by $\frac{\pi}{2}$ relative to the phase of the voltage (or the phase of the voltage is equally rushed relative to the current).

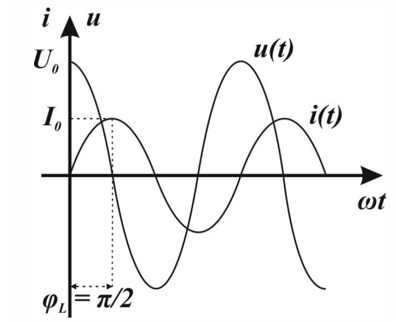

Figure 3.41

This phase-delaying effect of the coil (Figure 3.41) is a consequence of self-induction, the "inertia" of the current. As the current flowing through the coil increases from 0 to I_0, the alternating current in the coil generates a time-varying magnetic field. This induces a voltage at any moment that is opposite to the instantaneous voltage of the power source. This causes the resistance of the ideal coil to alternating current. At the maximum change in current, the voltage induced in the coil will be maximum. At the same time, the current increases only slowly due to Lenz's law. As the current decreases, the change in the magnetic field induces a voltage that is in the opposite direction. This delays the power outage. So in the presence of a coil, the current delays the voltage.

Ideally, the ohmic resistance of a coil is zero, so no heat is generated on it. Such a consumer who does not cause energy loss is called an inert consumer, its X_L resistance is called a barren resistance. In reality, the coil wire has an ohmic resistance, which results in heat generation, i.e. electrical energy loss.

3.4.5.1.3. Consumer with capacitive resistance

In a circuit containing a capacitor, direct electrical current only flows for a short time when the capacitor is charged because the capacitor's armature is an interruption to the current. However, in an AC circuit, the direction of the electric current changes every half period, causing the charges on the capacitor arms to swap. Therefore, alternating current may flow in such a circuit, however, there is still no charge flow between the armaments.

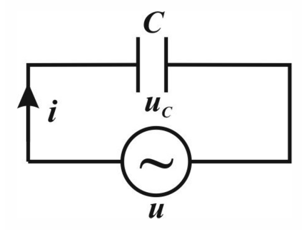

Figure 3.42

In a circuit containing a capacitor of capacity C (Figure 3.42), according to Kirchhoff's II law, the voltage of the capacitor is equal to the voltage of the current source:

$$u = u_C\,,$$

and using the $u = \frac{Q}{C}$ relationship, by the instantaneous voltage $u(t) = U_0 \sin(\omega t)$:

$$U_0 \sin(\omega t) = \frac{Q}{C}\,,$$

of which the instantaneous charge of the capacitor:

$$Q(t) = CU_0 \sin(\omega t).$$

The instantaneous value of the current flowing in the circuit is:

$$i = \frac{dQ(t)}{dt} = \frac{d}{dt}(CU_0 \sin\omega t) = C\omega U_0 \cos\omega t = C\omega U_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Then by introducing $I_0 = C\omega U_0$:

$$i(t) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right).$$

From the relation $I_0 = C\omega U_0$ we introduce the analogy of Ohm's law with

$$\boxed{X_C = \frac{1}{\omega C}}$$

capacitive resistance (capacitive reactance). Difference between phase angles of voltage and current:

$$\Delta\varphi_C = \omega t - \left(\omega t + \frac{\pi}{2}\right) = -\frac{\pi}{2}\,.$$

Based on this, it can be concluded that in the case of an inductive consumer, the phase of the current is rushed by $\frac{\pi}{2}$ relative to the phase of the voltage (or the phase of the voltage is equally delayed relative to the current, see Figure 3.43).

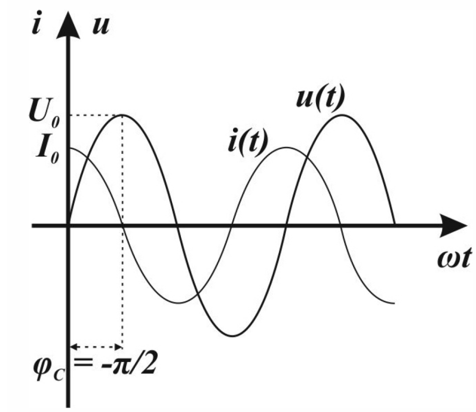

Figure 3.43

This is due to the fact that when the circuit is closed, the current charging the capacitor starts immediately, but the voltage of the capacitor only develops late in time. This phase-accelerating effect of the capacitor is due to the fact that when the circuit is closed, the charges flow unobstructed to the uncharged capacitor, a current of maximum strength flows in the conduction, while the voltage between the arms is zero. When the capacitor is charging, the voltage between the arms is maximal when the instantaneous value of the electric current charging the capacitor has become zero.

The capacitor is also a barren consumer, heat does not develop, X_C resistance is a barren resistance.

3.5. Electromagnetic vibrations

3.5.1. Free vibration of serial RLC circuit

Free vibration of a serial RLC circuit is generated if e.g. the electrical current generated in a series circuit consisting of an ohmic consumer of resistance R, a coil of inductance L and a capacitor of capacity C is vibrated by charging the capacitor and then closing the circuit (Figure 3.44):

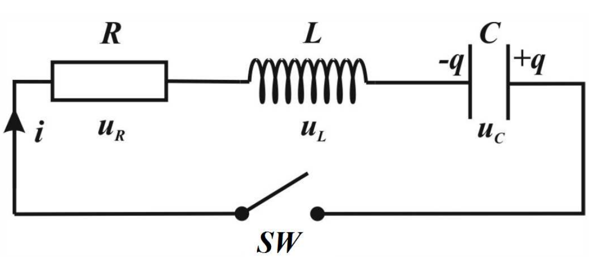

Figure 3.44

After closing switch SW, the capacitor initiates a discharge current in the circuit and according to Kirchhoff's II law (loop law):

$$u_R + u_L + u_C = Ri + L\frac{di}{dt} + \frac{Q}{C} = 0$$

This is a homogeneous differential equation with constant coefficients, where $i = \frac{dQ}{dt}$. The solution, apart from the mathematical detail:

$$\boxed{i = I_0 e^{-\frac{R}{2L}t} \sin \omega t},$$

that is, an exponentially damping sinusoidal vibration is generated (Figure 3.45):

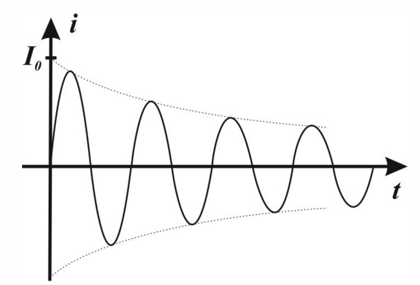

Figure 3.45

The reason for the attenuation is the heat loss on the resistor R. If $R = 0$, then

$$i = I_0 \sin \omega t,$$

that is, in the lossless state, an unattenuated vibration develops. Since $R = 0$ is an idealized state (think of the ohmic resistance of the coil), to generate unattenuated vibration, the RLC circuit must be supplied with energy from an external energy source in synchronism with the vibration, and an amplifier circuit must be used.

3.5.2. Forced vibration of serial RLC circuit

A forced oscillation of a series RLC circuit occurs when a series circuit consisting of an ohmic consumer of resistance R, a coil of inductance L and a capacitor of capacity C is connected to an alternating power source (Figure 3.46):

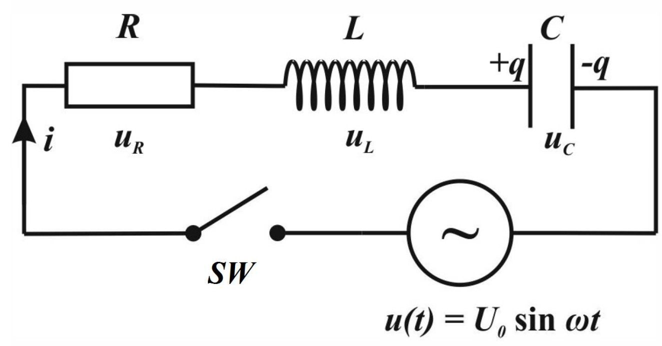

Figure 3.46

After closing switch SW, the capacitor initiates a discharge current in the circuit and according to Kirchhoff's II law (loop law):

$$u_R + u_L + u_C = u$$

where $u = U_0 \sin \omega t$ is the "forced voltage" and therefore the differential equation describing the electric current flowing in the circuit:

$$Ri + L\frac{di}{dt} + \frac{Q}{C} = U_0 \sin \omega t$$

Using $i = \dfrac{dQ}{dt}$ the solution apart from the mathematical detail:

$$\boxed{i = \frac{U_0}{\sqrt{R^2 + (X_L - X_C)^2}} \sin(\omega t + \varphi)},$$

where $\tan \varphi = \dfrac{X_L - X_C}{R}$ is the initial phase. Conclusions to be drawn from the solution:

1. The frequency of the current flowing in the circuit is equal to the frequency of the generator (the steady state of coercion).

2. The quantity $\sqrt{R^2 + (X_L - X_C)^2}$ is the resulting apparent resistance, or so-called impedance, of the circuit, denoted *Z*. Thus, we can conclude that the resistances (reactances) of the individual series-connected circuit elements do not add up algebraically, as in the DC networks, the series-connected ohmic consumers, but vectorically as shown in Figure 3.47:

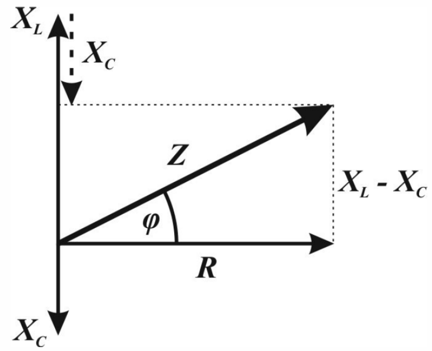

Figure 3.47

3.5.3. High frequency electromagnetic vibrations

The so-called technical alternating currents (vibrations) are produced in electromechanical generators ($f_{\max} \sim 300\ Hz$) that are rotated by mechanical energy. These alternating currents play an indispensable role in the supply of household and industrial electricity, and their voltage can be transformed up and down according to the goals of economical transportation and the needs of the consumer.

Of great importance for usability are electromagnetic vibrations from a few hundred *kHz* to a few hundred *GHz*, which can be generated by electronic devices without moving parts.

These vibrations are collectively referred to as radio frequency vibrations and are classified according to frequency:

- long waves:		$f <$	$300\ kHz$		$\lambda >$	$10^3\ m$
- medium waves:	$300\ kHz$	$< f <$	$3\ MHz$	$10^3\ m$	$> \lambda >$	$10^2\ m$
- short waves:	$3\ MHz$	$< f <$	$30\ MHz$	$10^2\ m$	$> \lambda >$	$10\ m$
- ultra short waves:	$30\ MHz$	$< f <$	$300\ MHz$	$10\ m$	$> \lambda >$	$1\ m$
- microwaves:	$300\ MHz$	$< f <$	$300\ GHz$	$1\ m$	$\lambda >$	$10^{-3}\ m$

These electromagnetic vibrations can be emitted into free space (air, vacuum) and propagate as transverse waves (see later). The wavelength range of radio waves thus spans about six orders of magnitude (1 mm to 1 km).

3.6. The shift current in the vacuum

By the 1870s, the laws of electricity and magnetism were known in the following forms:

Coulomb's law: $F_C = k\dfrac{Q_1 Q_1}{r^2}$

Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \dfrac{\sum Q}{\varepsilon_0}$

Lorentz's electromagnetic force law: $\vec{F}_L = Q\vec{E} + Q\vec{v} \times \vec{B}$

Amper's excitation law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J}_c \cdot d\vec{A}$

Faraday's law of induction $\mu_i = -\dfrac{d\Phi}{dt}$

These equations do not include a relationship between a time-varying electric field $\vec{E}(t)$ and a variable induction magnetic field $\vec{B}(t)$. The existence of electromagnetic waves does not conclude from the equations known at the time.

Maxwell recognized that Amper's excitation law is incomplete, leading in some cases to contradiction. In the absence of experimental evidence, it was theoretically concluded that the temporal change of an electric field of field strength $\vec{E}(t)$ produces a so-called shift current with a current density of:

$$J_s = \varepsilon_0 \frac{\partial \vec{E}(t)}{\partial t}.$$

The consequence of the shift current in the vacuum is that a magnetic field of induction $\vec{B}(t)$ is created around it. This assumption was only proved by the discovery of electromagnetic waves (Heinrich HERTZ, 1887).

With this Ampere's law of excitation, as supplemented by Maxwell, changed to shape:

$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \left(\vec{J}_c + \varepsilon_0 \frac{\partial \vec{E}(t)}{\partial t} \right) \cdot d\vec{A}}.$$

The above law is Equation IV of the electromagnetic theory united by Maxwell, which describes in a vacuum the so-called vortex strength of the magnetic field. The law is worded: “The magnetic field has vortex, the induction lines of the field are always self-closing curves.”

3.7. The system of Maxwell equations in vacuum

The electromagnetic theory combined by Maxwell is described by four equations that provide a solution to every electromagnetic problem and are now called Maxwell equations. These are:

I.

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{\sum Q}{\varepsilon_0}}$$

"The static electric field has source, the sources are the electric charges. The force lines of the static electric field start from the charges and go to infinity."

II.

$$\boxed{\oint \vec{E} \cdot d\vec{s} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}}$$

"A time-varying magnetic field induces a vortex electric field. The force lines of the induced electric field are self-closing curves."

III.

$$\boxed{\oint \vec{B} \cdot d\vec{A} = 0}$$

"The static magnetic field is source-free, no real magnetic charges (poles) exist. A closed surface always exits the same number of lines of force as it enters."

IV.

$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \left(\vec{J}_c + \varepsilon_0 \frac{\partial \vec{E}(t)}{\partial t} \right) \cdot d\vec{A}}$$

"The time-varying electric field as well as the conductive electric currents induce a vortex magnetic field. The induction lines of the magnetic field are self-closing curves."

3.8. Electromagnetic waves

Let's summarize the experience in advance:

1. If $\vec{J}_c = 0$ and $\vec{E}$ uniformly changes in Maxwell's IV equation, that is, $\frac{\partial \vec{E}(t)}{\partial t} = constant$, then a time-constant vortex magnetic field of induction $\vec{B}$ is created (Figure 3.48).

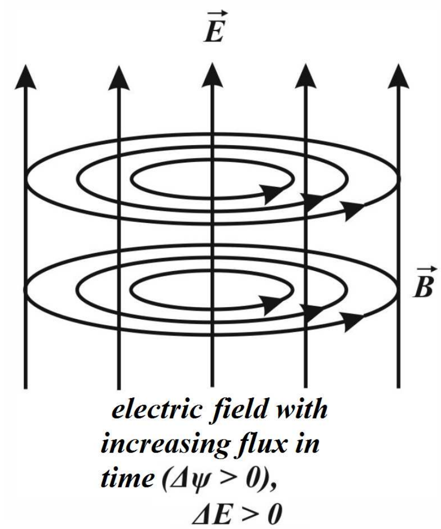

Figure 3.48

2. . If $\vec{B}$ is uniformly changes in Maxwell's II equation, that is, $\frac{\partial \vec{B}(t)}{\partial t} = constant$, then a time-constant vortex electric field of field strength $\vec{E}$ is created (Figure 3.49).

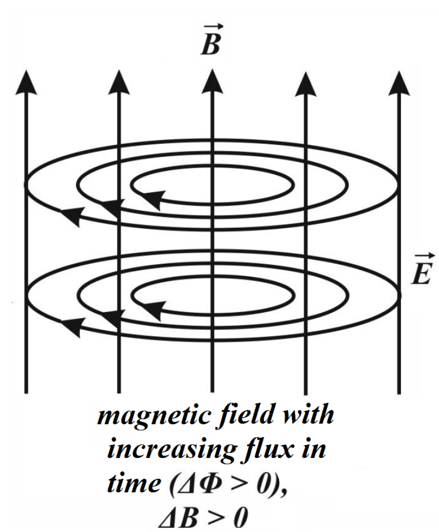

Figure 3.49

This suggests that an electric field that does not change uniformly over time creates a vortex magnetic field with time-varying induction, which in turn creates a vortex electric field with time-varying field strength, and so on. This effect is thus able to propagate in space through the induction of electric and magnetic fields.

From the Maxwell equations it can be deduced that the electric field induced by the time-varying magnetic field (apart from the derivation due to its complexity) is in the simplest case in one dimension the

$$\boxed{\frac{\partial^2 E_y}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}}$$

describes a partial differential equation, called a wave equation, where $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \cdot 10^8 \frac{m}{s}$ is the speed of light.

This is mathematically identical to the wave equation described in Mechanics and its solution is:

$$\boxed{E_y(x,t) = E_{y,0} \sin \omega\left(t - \frac{x}{c}\right)}.$$

Aware of this, Maxwell came to the conclusion that there must be electromagnetic waves detaching from the center of vibration and moving freely at the speed of light. This thought led to the light being an electromagnetic wave.

3.8.1. Generation of electromagnetic radio waves

Electromagnetic waves can be generated in a number of ways, but each rests on the principle that radiation is emitted by accelerating charges.

We have previously seen that in a closed resonant circuit, the electric and magnetic fields are localized in the capacitor and in the coil, respectively, and no vibrational energy is radiated. However, if the spatial geometry of the resonant circuit consisting of the coil and the capacitor is modified as shown in Figure 3.50, the electric and magnetic fields extend over a larger range, the electric field lines outside the capacitor at the opposite charge arm, and field is induced perpendicular to the electric power lines. A change in the induced magnetic field over time induces a vortex electric field in planes perpendicular to the induction lines, and so on. According to the Maxwell equations, the time-varying fields inducing each other move away from the vibrating center (antenna) at the speed of light.

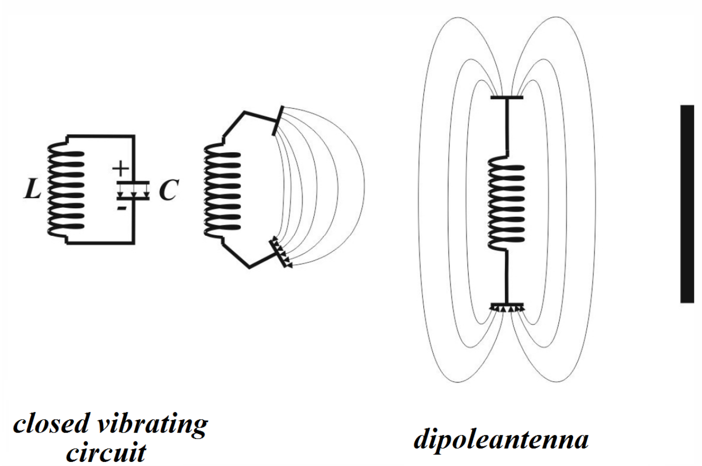

Figure 3.50

Of course, the radiated energy gradually reduces the energy in the open LC resonant circuit (antenna), and when the oscillation ceases, the radiation also ceases. The length of the antenna emitting the waves is half the wavelength of the generated electromagnetic wave $\left(\frac{\lambda}{2}\right)$. During vibration, the ends (the armor of the capacitors) are alternately positively and negatively charged, therefore its called dipoleantenna, the electromagnetic waves emitted by such an antenna are called dipole radiation.

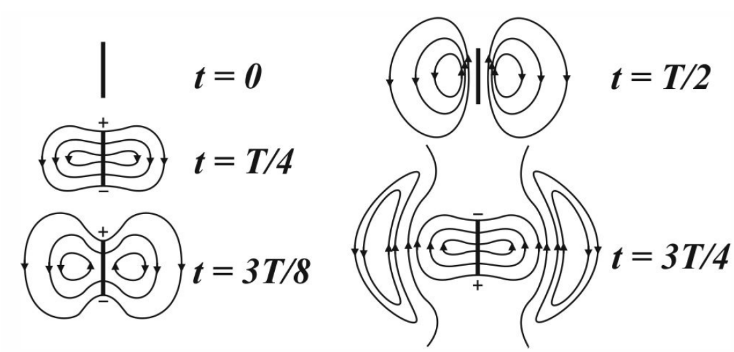

Figure 3.51

Examine the propagation of the wave along a given direction, which should be the x-direction perpendicular to the longitudinal axis of the antenna. Then the vibration of the electric field strength $\vec{E}$ occurs in the y direction, while that of the magnetic induction $\vec{B}$ occurs in the z direction, so the wavefunctions describing their propagation in the x direction are:

$$\boxed{E_y(x,t) = E_{y,0}\sin\omega\left(t - \frac{x}{c}\right)},$$

and

$$\boxed{B_z(x,t) = B_{z,0} \sin \omega \left(t - \frac{x}{c} \right)}.$$

We can see that at a given location in the dipole radiation field, the magnitude of the vectors $\vec{E}$ and $\vec{B}$ is described by harmonics oscillating perpendicular to each other and to the direction of wave propagation, which are in the same phase with each other.

The condition for the constant power radiation of the dipole antenna is that an energy supply must be provided, which is possible by connecting an alternating voltage power supply (oscillator). Figure 3.52 illustrates the lines of force of an electric field generated around a dipole antenna with constant radiated power and moving away at the speed of light, and the magnetic induction lines perpendicular to the axis of the antenna induced by them.

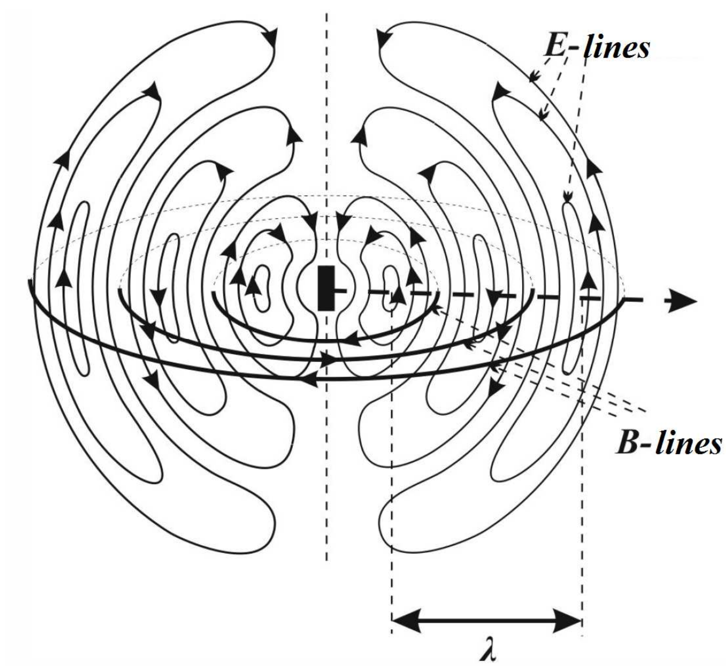

Figure 3.52

The tangents of the y-direction electric force lines and the z-direction magnetic induction lines perpendicular to the antenna axis in the direction perpendicular to the antenna axis plot the waveform of the sinusoidal electromagnetic vibrations (Figure 3.53):

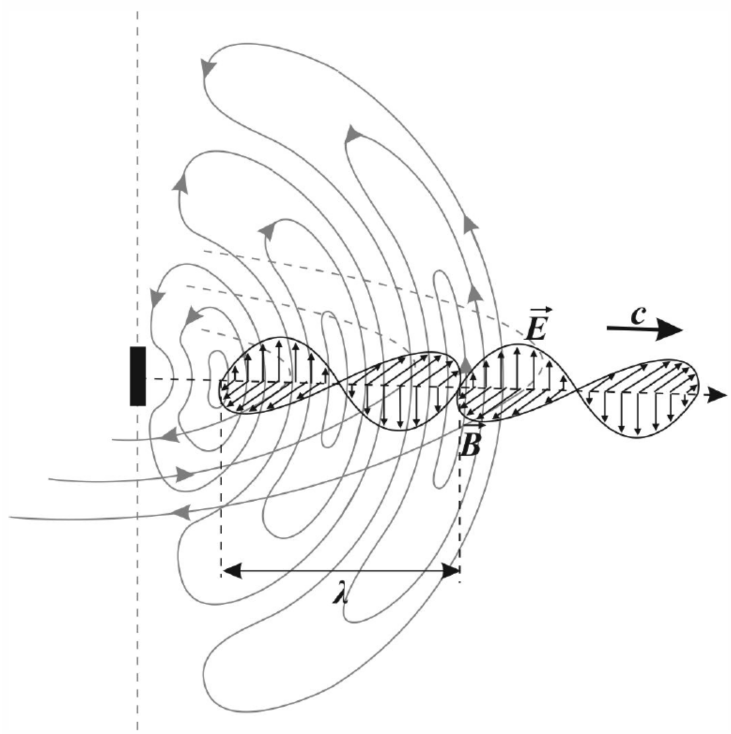

Figure 3.53

3.8.2. Energy density of electromagnetic waves

The vibrational energy propagating in an electromagnetic wave is characterized numerically by the vibrational energy passing through the unit surface per unit time, or the so-called surface power density:

$$S = \frac{1}{\mu_0} E_y B_z .$$

The sign of the surface power density is: *S*, its unit in SI: $\frac{W}{m^2}$. The surface power density vector, also known as the Poynting-vector, is obtained as the vector product of the vectors $\vec{E}$ and $\vec{B}$:

$$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}},$$

whose magnitude is the surface power density, the direction of which is the same as the direction of wave propagation. Using the sinusoidally alternating values of E_y and B_z, the absolute value of the Poynting vector $\vec{S}$ is:

$$S = \frac{1}{\mu_0} E_{y,0} B_{z,0} \sin^2 \omega\left(t - \frac{x}{c}\right),$$

which means it reaches its maximum twice in a period of time.

3.8.3. Applications of radio waves

Except in some cases, radio waves are used to transmit information. To do this, the electrical signal carrying the information to be transmitted (eg sound, image) must be superimposed on the radio wave, ie the radio wave must be converted into an information carrier. The information to be transmitted is applied by modulation to the high frequency carrier wave, a characteristic of which is changed by the signal containing the information. That's what we're talking about

- Amplitude modulation, when the amplitude of the carrier vibration is changed according to the information to be transmitted.
- Frequency modulation, when the frequency of the carrier vibration is changed according to the information to be transmitted.
- Phase modulation, when the phase of the carrier vibration is changed according to the discretely changing information to be transmitted.

The use of radio waves in imaging and therapy is of paramount importance in medical applications. In nuclear magnetic resonance imaging, the interaction of hydrogen nuclei with a magnetic field provides information on the water content and water movement of tissues by analyzing the radiation of ultrashort wavelength radio waves. The effect of the microwave range based on heat generation is used for radiotherapy to treat inflammation.

There are also electromagnetic waves with a shorter wavelength (higher frequency) than radio waves, such as light. Their origin, nature and their interactions will be discussed in the section on Nuclear Physics.

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